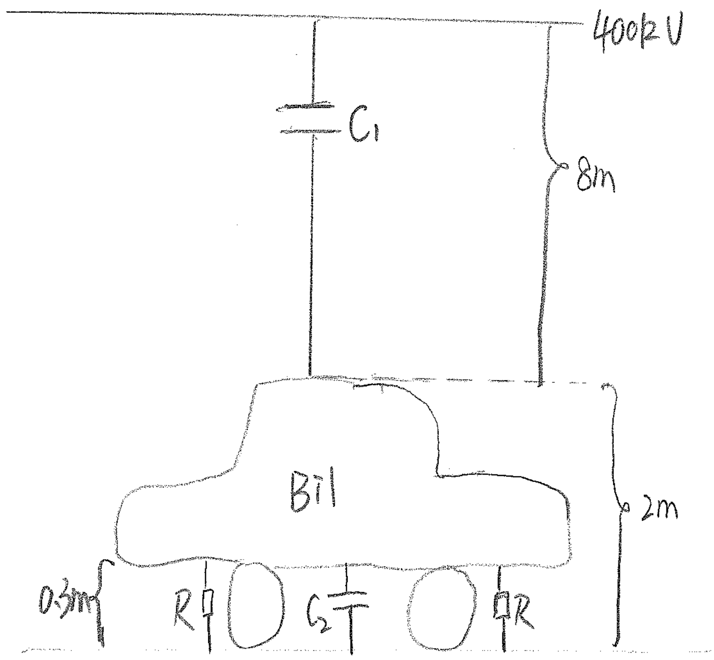
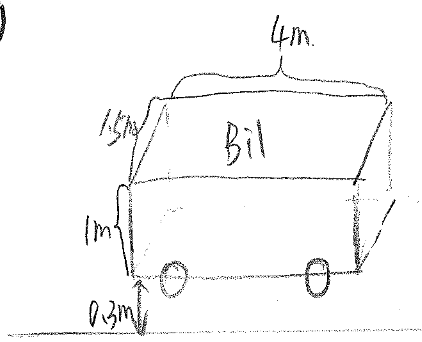
2008-01-17

Problem #1

a)



b)



top roof model of the car

$A_{bil} = A_{top} +$ "edge effect"

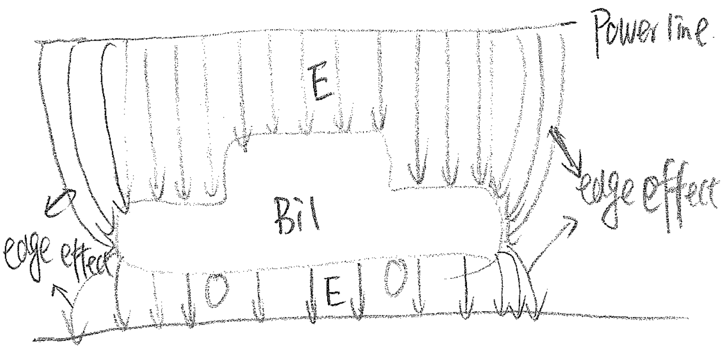
$\approx 10m^2$

$d_1 = 8m$

$d_2 = 0.3m.$

$C = \frac{\varepsilon A}{d} \Rightarrow C_1 = \frac{\varepsilon_0 \cdot 10}{8} F \approx 11 pF$

$C_2 = \frac{\varepsilon_0 \cdot 10}{0.3} F \approx 0.30 nF$



R=0. good isolator.

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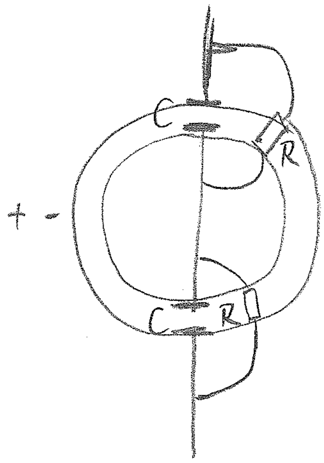
Problem #2

③

a)



b) Cell diameter $D = 50\mu m$, membrane thickness $d = 10nm$. Voltage across membrane: $U = 70mV$

$$C_{membrane} = \frac{\varepsilon A}{d} = \frac{10^6 \varepsilon_0 \cdot \left(\frac{50 \cdot 10^{-6}}{2}\right)^2 \pi}{10 \cdot 10^{-9}} \approx 1.74 \mu F$$

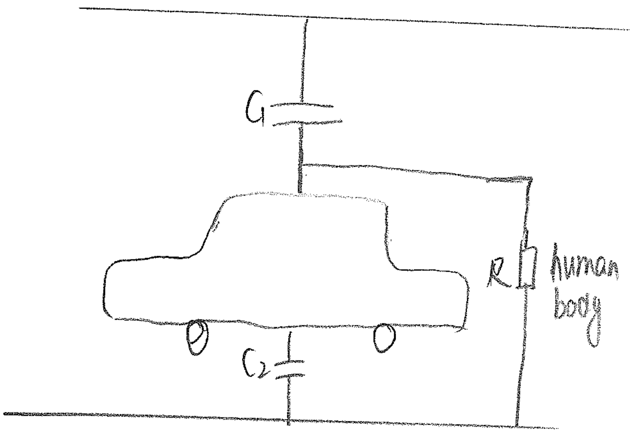
$$R_{membrane} = \frac{d}{\sigma A} = \frac{10 \cdot 10^{-9}}{0.1 \left(\frac{50 \cdot 10^{-6}}{2}\right)^2 \pi} \approx 50.9 \Omega$$

c) $$Z_{total} = \frac{1}{1/R_{membrane} + j\omega C_{membrane}} \approx 50.86 - j1.445 (\Omega)$$

$$|I| = \frac{|U|}{|Z|} = \frac{70 \cdot 10^{-3}}{|Z_{total}|} = \frac{70 \cdot 10^{-3}}{\sqrt{50.86^2 + 1.445^2}} \approx 1.3758 \cdot 10^{-3} A \approx 1.3758 mA$$

d) ① the liquid is not perfect conductor. and due to the finite conductivity. there must be some loss there.

② the model we used is based on rectangular geometry. while the cell is not rectangular shaped c)



$\sigma = 0.2\,S/m$

Suppose $f = 50Hz$.

$$R = \frac{l}{A\sigma} \simeq \frac{1.7}{0.3 \cdot 0.3 \cdot 0.2} = 100\Omega$$

$$Z_{total} = \frac{1}{j\omega C_1} + \frac{1}{\frac{1}{1/j\omega C_2} + \frac{1}{R_{body}}} = -j\,2.88 \cdot 10^8 + \frac{1}{j\,9.27 \cdot 10^{-8} + 0.01}$$

($j\,9.27 \cdot 10^{-8}$ → very small)

$$\simeq -j\,2.88 \cdot 10^8 + \frac{1}{0.01}$$

($\frac{1}{0.01}$ → very small also)

$$\simeq -j\,2.88 \cdot 10^8$$

$$I_{body} = \frac{V}{Z_{total}} = \frac{V}{\frac{1}{j\omega C_1}} = j\omega \varepsilon_0 \frac{A}{d} V = j\omega \varepsilon_0 A E$$

$$E = \frac{V}{d} = \frac{400kV}{8m} = 50kV/m$$

$$A = 10m^2$$

$$|I_{body}| = \omega \varepsilon_0 A E = 2\pi \cdot 50 \cdot \varepsilon_0 \cdot 10 \cdot 50 \times 10^3 \simeq 1.4mA$$

d) NO danger. It becomes dangerous when it is more than 5mA

2008-01-17 (2007-06-02 Problem #3)

Problem #3

$$U_2 = \frac{R_1}{R_1 + \frac{1}{jwC}} U_1$$

$$\Rightarrow \frac{U_2}{U_1} = \frac{R_1}{R_1 + \frac{1}{jwC}} = \frac{R_1}{R_1 - \frac{j}{wC}} = \frac{R_1(R_1 + \frac{j}{wC})}{(R_1^2 + \frac{1}{w^2C^2})} = \frac{R_1^2 + j\frac{R_1}{wC}}{R_1^2 + \frac{1}{w^2C^2}}$$

$$\Rightarrow \frac{U_2}{U_1} = \frac{w^2C^2R_1^2 + jwCR_1}{1 + w^2C^2R_1^2}$$

$$\Rightarrow |H(w)| = \sqrt{\frac{w^2C^2R_1^2}{(1+w^2C^2R_1^2)^2} + \frac{w^4C^4R_1^4}{(1+w^2C^2R_1^2)^2}}$$

$$= \frac{wCR_1\sqrt{1+w^2C^2R_1^2}}{1+w^2C^2R_1^2}$$

$$\frac{wCR_1\sqrt{1+w^2C^2R_1^2}}{1+w^2C^2R_1^2} = \frac{1}{\sqrt{2}} \Rightarrow \frac{w^2C^2R_1^2(1+w^2C^2R_1^2)}{(1+w^2C^2R_1^2)^2} = \frac{1}{2}$$

$$\Rightarrow \frac{1+w^2C^2R_1^2}{2} = w^2C^2R_1^2$$

$$\Rightarrow w^2C^2R_1^2 = 1$$

$$\Rightarrow w^2 = \frac{1}{C^2R_1^2}$$

$$\Rightarrow w = \frac{1}{CR_1} \Rightarrow f_u = \frac{1}{2\pi CR_1}$$

$$f_u = \frac{1}{2\pi \cdot 4.7 \cdot 10^{-9} R_1} = 100$$

$$\Rightarrow R_1 = \frac{1}{2\pi \cdot 4.7 \cdot 10^{-9} \cdot 100} = 3.3 \cdot 10^5 \Omega$$

$$f_ö = \frac{1}{2\pi \cdot 4.7 \cdot 10^{-9} R_2} = 4 \cdot 10^3$$

$$\Rightarrow R_2 = \frac{1}{2\pi \cdot 4.7 \cdot 10^{-9} \cdot 4 \cdot 10^3} = 8.5 \cdot 10^3 \Omega$$

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(5)

Problem #4

Capacitor impedance is 200Ω. what is the Capacitance?

$$Z_C = \frac{1}{j\omega C} \qquad |Z_C| = 200\Omega$$

$$\Rightarrow \left|\frac{1}{j\omega C}\right| = 200 \Rightarrow \frac{1}{\omega C} = 200 \Rightarrow C = \frac{1}{\omega 200}$$

What is w? Current is given as $i = 0.05\sin(100\pi t)A \Rightarrow \omega = 100\pi$

$$C = \frac{1}{100\pi \cdot 200} = 15.9\mu F$$

$$U_1 = 5.0\sin(100\pi t)V$$

$$U_2 = 10.0\sin(100\pi t - \frac{\pi}{2})V$$

The input voltage:

$$\begin{aligned} U_{in} &= U_1 + U_2 \\ &= 5.0\sin(100\pi t) + 10.0\sin(100\pi t - \frac{\pi}{2}) \\ &= 5.0\sin(100\pi t) - 10.0\cos(100\pi t) \\ &= \sqrt{5.0^2 + 10.0^2}\sin(100\pi t + \text{atan}\frac{-10.0}{5}) \\ &\simeq 11.2\sin(100\pi t - 1.1) \end{aligned}$$