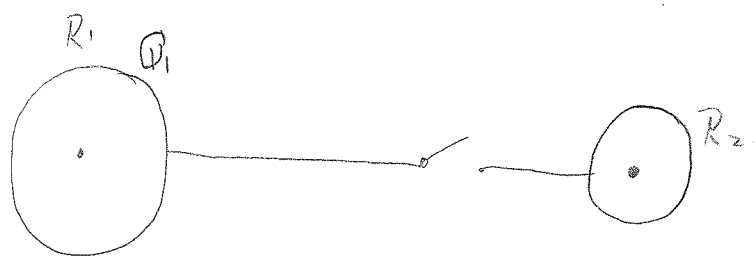


① 2009-01-05 Problem 1



a) after the switch is closed the potential on the two metal balls should be the same. Suppose that the charge on ball 1 is  $Q_1'$ . Then the charge on ball 2 should be  $Q_1 - Q_1'$ .

$$\frac{Q_1'}{4\pi R_1} = \frac{Q_1 - Q_1'}{4\pi R_2}$$

3 points

$$\frac{Q_1'}{Q_1 - Q_1'} = \frac{R_1}{R_2}$$

$$R_2 Q_1' = R_1 Q_1 - R_1 Q_1'$$

$$(R_1 + R_2) Q_1' = R_1 Q_1$$

$$Q_1' = \frac{R_1 Q_1}{R_1 + R_2}$$

$$Q_2 = Q_1 - Q_1' = Q_1 - \frac{R_1 Q_1}{R_1 + R_2} = \frac{R_2 Q_1}{R_1 + R_2}$$

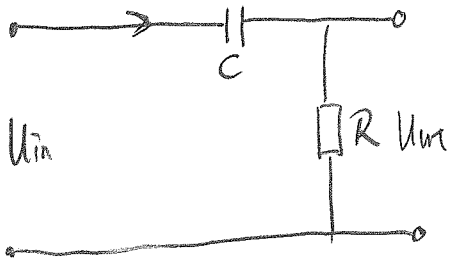
b) if the Gauss law is used, then the conductor should be long enough to make sure the field from this two balls are not interfered too much with each other

c) As far as the distance between these two balls are much larger compared to the radius of the ball and the connecting conductor has a much smaller diameter. the result is a good estimation.

d) Yes. It has some similarity to lightning conductor (thunder). The larger metal ball can be a electronic cloud in the sky and the building on the ground is the smaller metal ball

2009-01-15 Problem 2

Describe what the circuit is used for? and calculate how  $U_{out}$  dependent of  $U_{in}$ ? What happen if change the position of resistance and capacitance?



$$\bar{U}_{out} = \bar{i} R = \frac{\bar{U}_{in}}{\bar{Z}_R + \bar{Z}_C} = \frac{\bar{U}_{in} \cdot R}{R + \frac{1}{j\omega C}}$$

$$H(j\omega) = \frac{\bar{U}_{out}}{\bar{U}_{in}} = \frac{R}{R + \frac{1}{j\omega C}}$$

$$|H(j\omega)| = \frac{R}{\sqrt{R^2 + (\frac{1}{\omega C})^2}}$$

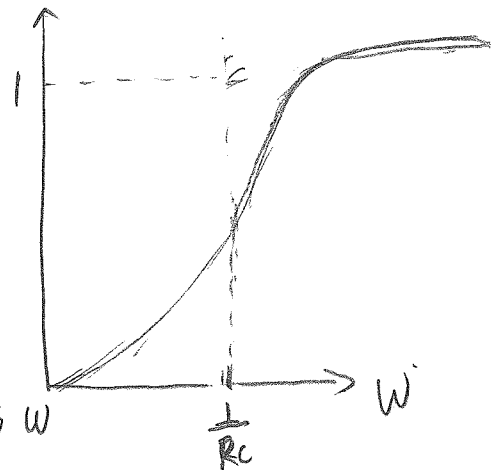
$$= \frac{RC}{\sqrt{(RC)^2 + (\frac{1}{\omega})^2}}$$

when  $\frac{1}{\omega} \ll RC \Rightarrow \omega \gg \frac{1}{RC}$

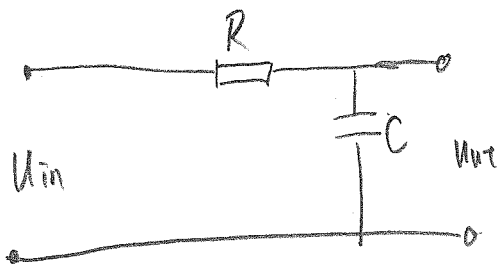
$$|H(j\omega)| \approx \frac{RC}{\sqrt{(RC)^2}} = 1$$

When  $\frac{1}{\omega} \gg RC \Rightarrow \omega \ll \frac{1}{RC}$

$$|H(j\omega)| \approx \frac{RC}{\frac{1}{\omega}} = \omega RC \text{ increase as } \omega$$



High Pass filter



$$H(j\omega) = \frac{\bar{U}_{out}}{\bar{U}_{in}} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

$$|H(j\omega)| = \frac{\frac{1}{\omega C}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$$

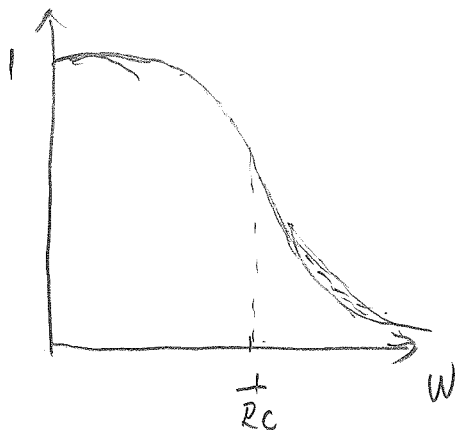
$$= \frac{1}{\sqrt{(\omega RC)^2 + 1}}$$

When  $\omega RC \gg 1$   $\omega \gg \frac{1}{RC}$

$|H(j\omega)| \approx \frac{1}{\omega RC}$  decrease as  $\omega$

When  $\omega RC \ll 1$   $\omega \ll \frac{1}{RC}$

$|H(j\omega)| \approx 1$



Low pass filter