Problem 1

a) Gauss law

$$\oint_S \vec{E}\cdot d\vec{s} = \frac{Q}{\varepsilon_0} \Rightarrow E\cdot 4\pi R^2 = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E = \frac{Q}{4\pi\varepsilon_0 R^2}$$

When $R<a$

$$Q=0 \Rightarrow E=0$$

When $a\le R<b$

$$Q=Q_1 \Rightarrow E=\frac{Q_1}{4\pi\varepsilon_0 R^2}$$

When $R\geq b$

$$Q=Q_1+Q_2 \Rightarrow E=\frac{Q_1+Q_2}{4\pi\varepsilon_0 R^2}$$

b) Connect two metal shells with a conductor, the charge will redistribute until the two metal shells have the same potential.

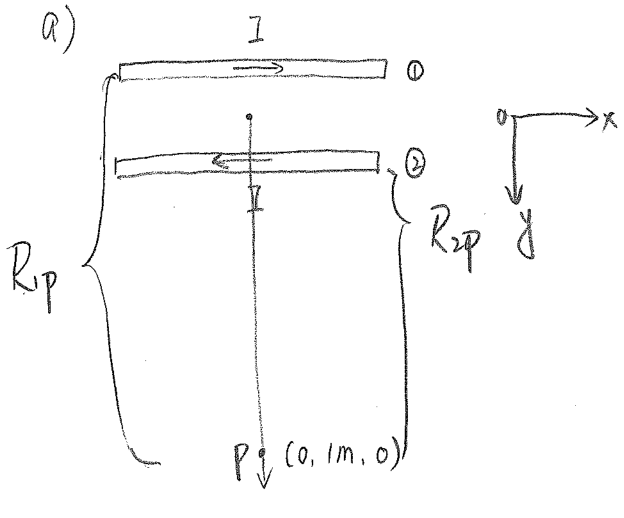
$R<a$ $\quad Q=0 \Rightarrow E=0$

$a\le R<b$ $\quad E=0$ [There is no E-field in the region between two equal potential surfaces]

$R\geq b$ $\quad Q=Q_1+Q_2 \Rightarrow E=\dfrac{Q_1+Q_2}{4\pi\varepsilon_0 R^2}$

Program 2

a)



$$\vec{B}_P = \frac{\mu_0 I}{2\pi R_{1P}}\hat{z} - \frac{\mu_0 I}{2\pi R_{2P}}\hat{z}$$

$$= \frac{4\pi\cdot 10^{-7}\cdot 2}{2\pi\left(1m+\frac{1cm}{2}\right)}\hat{z} - \frac{4\pi\cdot 10^{-7}\cdot 2}{2\pi\left(1m-\frac{1cm}{2}\right)}\hat{z}$$

$$= \frac{4\times 10^{-7}}{(1+0.005)}\hat{z} - \frac{4\times 10^{-7}}{(1-0.005)}\hat{z}$$

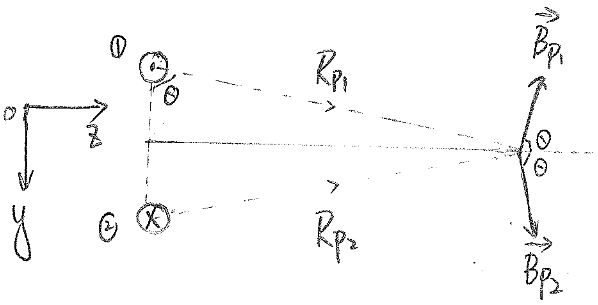
$$= 4\times 10^{-7}\hat{z}\left[\frac{1}{1.005} - \frac{1}{0.995}\right]$$

$$= 4\times 10^{-7}\hat{z}(-0.01)$$

$$= -4\times 10^{-9}\hat{z}\ T$$

Magnitude: 4×10^{-9} T Direction: negative z

b)



$$\vec{B}_{P1} = \frac{\mu_0 I}{2\pi R_{P1}}\left[\cos\theta\,\hat{z} - \sin\theta\,\hat{y}\right]$$

$$\vec{B}_{P2} = \frac{\mu_0 I}{2\pi R_{P2}}\left[\cos\theta\,\hat{z} + \sin\theta\,\hat{y}\right]$$

$$R_P = R_{P1} = R_{P2} = \sqrt{(0.05)^2+(0.005)^2} \approx 0.0502\,m$$

$$\tan\theta = \frac{5cm}{0.5cm} = 10$$

$$\cos\theta = \frac{1}{\sqrt{1+\tan^2\theta}} = \frac{1}{\sqrt{101}}$$

$$\vec{B}_P = \vec{B}_{P1} + \vec{B}_{P2} = \frac{\mu_0 I}{2\pi R_P}\cdot 2\cos\theta\,\hat{z} = \frac{4\pi\cdot 10^{-7}\cdot 2}{2\pi\cdot 0.0502}\cdot 2\cdot \frac{1}{\sqrt{101}}\hat{z}$$

$$\approx 1.59\times 10^{-6}\hat{z}\ T$$

Magnitude: 1.59×10^{-6} T Direction: Positive z

Program 3

a) when $U = 10V$

$$I_{RA} = \frac{U}{R_A} = \frac{10V}{50\Omega} = 0.2A$$

$U = 10V$ is a direct current voltage. so the frequency $\omega = 0$. Then the impedance of C is: $Z_C = \frac{1}{\omega C} = \infty$ is open circuit and no current through the branch: $I_{RB} = 0$

$\omega = 0 \Rightarrow Z_L = \omega L = 0$ short circuit.

$$I_{RC} = \frac{U}{R_C} = \frac{10V}{50\Omega} = 0.2A$$

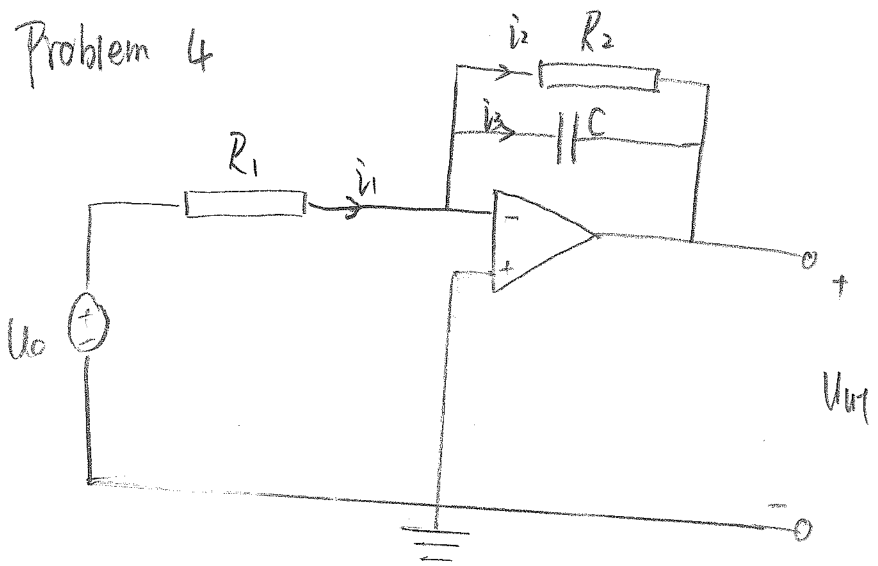
b) $U = 10\sin 100\pi t \ V \Rightarrow U_{eff} = \frac{10}{\sqrt{2}}$.

$$I_{RA}^{eff} = \frac{U_{eff}}{R_A} = \frac{10}{50\sqrt{2}} = \frac{1}{5\sqrt{2}} \approx 0.1414\ A$$

$$I_{RB}^{eff} = \frac{U_{eff}}{\sqrt{R_B^2 + \left(\frac{1}{\omega C}\right)^2}} = \frac{10}{\sqrt{2}} \cdot \frac{1}{\sqrt{50^2 + \left(\frac{1}{100\pi \cdot 10 \cdot 10^{-6}}\right)^2}} \approx 0.0219\ A$$

$$I_{RC}^{eff} = \frac{U_{eff}}{\sqrt{R_C^2 + (\omega L)^2}} = \frac{10}{\sqrt{2}} \cdot \frac{1}{\sqrt{50^2 + (100\pi \cdot 5 \cdot 10^{-3})^2}} \approx 0.1414A$$

Problem 4



$$\begin{cases} i_1 R_1 - U_0 = 0 \Rightarrow i_1 = \dfrac{U_0}{R_1} \\ i_2 R_2 + U_{ut} = 0 \Rightarrow i_2 = -\dfrac{U_{ut}}{R_2} \\ i_3 \cdot \dfrac{1}{j\omega C} + U_{ut} = 0 \Rightarrow i_3 = -j\omega C \cdot U_{ut} \\ i_1 = i_2 + i_3 \Rightarrow \dfrac{U_0}{R_1} = -\dfrac{U_{ut}}{R_2} - j\omega C\, U_{ut} \end{cases}$$

$$\Rightarrow U_{ut}\left(-\frac{1}{R_2} - j\omega C\right) = \frac{U_0}{R_1}$$

$$U_{ut} = \frac{U_0}{R_1} \cdot \frac{1}{\left(-\frac{1}{R_2} - j\omega C\right)}$$

$$= \frac{U_0}{-\frac{R_1}{R_2} - j\omega R_1 C}$$

$$= \frac{U_0 R_2}{-R_1 - j\omega R_1 R_2 C}$$

$$= -\frac{U_0 R_2}{R_1(1 + j\omega R_2 C)}$$

Program 5

a) From image 1 to image 2, we can find that dark points become brighter and bright points become even brighter. Therefore (c) is the correct histogram mapping.

b) The output image matrix

0	1	0
1	1	1
0	1	0

c) $g(x,y) = f(x,y) * h(x,y)$

$$= \sum_{a=-1}^{1} \sum_{b=-1}^{1} f(a,b)\, h(x-a, y-b)$$

midpoint: $g(0,0) = f(-1,-1)\,h(1,1) + f(-1,0)\,h(1,0) + f(-1,1)\,h(1,-1) +$
$f(0,-1)\,h(0,1) + f(0,0)\,h(0,0) + f(0,1)\,h(0,-1) +$
$f(1,-1)\,h(-1,1) + f(1,0)\,h(-1,0) + f(1,1)\,h(-1,-1)$

$= 1\cdot 0 + 1\cdot 1 + 1\cdot 0 + 1\cdot 1 + 6\cdot(-4) + 1\cdot 1 + 1\cdot 0 + 1\cdot 1 + 1\cdot 0$

$= 1 + 1 - 24 + 1 + 1$

$= -20$