Problem 1

According to Gauss law

(a) $$\oint_S \vec{E} \cdot d\vec{s} = \frac{Q_T}{\varepsilon_0}$$

$$E_r \cdot 4\pi r^2 = \frac{Q_T}{\varepsilon_0}$$

$$E_r = \frac{Q_T}{4\pi r^2 \varepsilon_0}$$

When $r < a$. $\quad Q_T = 0.$ $\quad E_r = 0$

When $r = a$. $\quad Q_T = Q$ $\quad E_r = \frac{Q}{4\pi a^2 \varepsilon_0}$

When $r > a$ $\quad Q_T = Q$ $\quad E_r = \frac{Q}{4\pi r^2 \varepsilon_0}$

(b) The potential of the sphere can be calculated as:

$$U = \int_a^\infty \vec{E} \cdot d\vec{l} = \int_a^\infty E_r \, dr$$

$$= \int_a^\infty \frac{Q}{4\pi \varepsilon_0 r^2} \, dr$$

$$= -\frac{Q}{4\pi \varepsilon_0 r}\Big|_a^\infty = \frac{Q}{4\pi \varepsilon_0 a}$$

Problem 2

(a) The B field at 1.0m from the double conductor's center is the sum up of the B field produced by each conductor. Since the conductor is assumed to be very long. the B field can be calculated as:

$$B_z = \frac{\mu_0 I}{2\pi r}$$

The B fields produced by the upper and lower conductors are respectively:

$$B_z^{upper} = \frac{\mu_0 I}{2\pi r_1} = \frac{\mu_0 \, 2.0 \sin(100\pi t)}{2\pi \cdot (1+0.004)}$$

$$B_z^{lower} = -\frac{\mu_0 I}{2\pi r_2} = \frac{\mu_0 \cdot 2.0 \sin(100\pi t)}{2\pi (1-0.004)}$$

The total B field is:

$$B_z^T = B_z^{upper} + B_z^{lower} = \frac{\mu_0 \cdot 2.0 \sin(100\pi t)}{2\pi} \left[\frac{1}{1.004} - \frac{1}{0.996} \right]$$

$$= -3.2 \cdot 10^{-9} \sin(100\pi t) \text{ T}$$

b) According to the same principle, the B field at the center point is:

$$B_z^T = \frac{\mu_0 I}{2\pi r_1} + \frac{\mu_0 I}{2\pi r_2} = \frac{\mu_0 I}{\pi r} = \frac{\mu_0 \cdot 2.0 \sin(100\pi t)}{\pi \cdot 0.004}$$

$$= 2.0 \cdot 10^{-4} \sin(100\pi t) \text{ T}$$

Problem 3

(a) The mean power over R can be calculated as:

$$P = \frac{\langle u^2 \rangle}{R}$$

According to the figure, the voltage is a triangle form with period of T, then u^2 is a signal with period of $\frac{T}{2}$.

The voltage from 0 to $\frac{T}{2}$ is a linear function of t and can be expressed as

$$u = at + b$$

When $t=0$, $u = b = -4V$

When $t = 0.5T$, $u = 0.5Ta + b = 4V \Rightarrow a = \frac{16}{T}$

Therefore $$u = \frac{16}{T}t - 4$$

Mean power:

$$P = \frac{\langle u^2 \rangle}{R} = \frac{1}{50} \cdot \frac{2}{T} \int_0^{\frac{T}{2}} \left(\frac{16}{T}t - 4\right)^2 dt$$

$$= \frac{1}{50} \cdot \frac{2}{T} \int_0^{\frac{T}{2}} \left(\frac{t^2}{T^2} \cdot 256 - 128\frac{t}{T} + 16\right) dt$$

$$= \frac{1}{50} \cdot \frac{2}{T} \left[\frac{256}{3T^2}t^3 - \frac{128}{2T}t^2 + 16t\right] \Bigg|_{t=0}^{t=\frac{T}{2}}$$

$$= \frac{8}{75} W$$

(b)

$$u = 10\sin(6\cdot10^{9}\pi t)$$

$$\omega = 6\cdot10^{9}\pi$$

$$i_{eff} = \frac{1}{\sqrt{2}}\frac{\hat{u}}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}} \approx 0.1414\,A \quad (\omega C = 6\cdot10^{9}\pi\cdot500\cdot10^{-12} = 3\pi)$$

$$P_R = i_{eff}^2\cdot R \approx 1{,}0\,W$$

Problem 4

(a)

$$\begin{cases} -U_0 + iR + U_c + U_{ut} = 0 \\ -U_0 + iR = 0 \\ i = C\dfrac{dU_c}{dt} \end{cases} \Rightarrow \quad U_c = -U_{ut} \Rightarrow i = -C\frac{dU_{ut}}{dt}$$

$$-U_0 + \left(-C\frac{dU_{ut}}{dt}\right)R = 0$$

$$\frac{dU_{ut}}{dt} = -\frac{U_0}{RC}$$

$$U_{ut} = -\int_{-\infty}^{t} \frac{U_0}{RC}\, dt.$$

or

$$\begin{cases} -U_0 + iR + \dfrac{1}{j\omega C}\cdot i + U_{ut} = 0 \\ -U_0 + iR = 0 \end{cases} \Rightarrow \quad i = -U_{ut}\cdot(j\omega C)$$

$$\Rightarrow -U_0 + (-U_{ut}\cdot j\omega C)R = 0$$

$$\Rightarrow U_{ut} = -\frac{U_0}{j\omega RC}.$$

(b) This circuit can be used as an integrator

Problem 5

(a)

From Image1 to image2, the bright part becomes brighter and. the dark part becomes darker. So the histogramapping (a) is used for the image1 changing to image 2.

(b) the output image is

$$\begin{matrix} 0 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{matrix}$$

c) the Value in the middle of the output matrix is:

$$g(x,y) = f(x,y) * h(x,y) = \sum_{a=-1}^{1} \sum_{b=-1}^{1} f(a,b)\, h(x-a, y-b)$$

$$g(0,0) = f(-1,-1)\,h(1,1) + f(-1,0)\,h(1,0) + f(-1,1)\,h(1,-1) +$$

$$f(0,-1)\,h(0,1) + f(0,0)\,h(0,0) + f(0,1)\,h(0,-1) +$$

$$f(1,-1)\,h(-1,1) + f(1,0)\,h(-1,0) + f(1,1)\,h(-1,-1)$$

$$= 2\cdot 0 + 1\cdot 1 + 2\cdot 0 + 1\cdot 1 + 4\cdot(-6) + 1\cdot 1 + 3\cdot 0 + 1\cdot 1 + 3\cdot 0$$

$$= 4 - 24$$

$$= -20$$