Problem 1

a) use Gauss law

$$\oint_S \vec{E}\cdot d\vec{s} = \frac{q}{\varepsilon_0} \Longrightarrow E_r = \frac{q}{4\pi\varepsilon_0 R^2}$$

when $R < a$ $\quad q = 0 \quad E_r = 0$

(Inside the sphere)

when $R \geq a$ $\quad q = Q \quad E_r = \frac{Q}{4\pi\varepsilon_0 R^2}$

b) Use a very long conductor connect another uncharged sphere to the sphere, then both spheres have the same potential Assume that after connection the charges on sphere 1 and sphere 2 are Q_1 and Q_2 respectively then

$$\left.\begin{aligned} Q_1 + Q_2 &= Q \\ \frac{Q_1}{4\pi\varepsilon_0 a} &= \frac{Q_2}{4\pi\varepsilon_0 b} \end{aligned}\right\} \Longrightarrow \begin{cases} Q_1 + Q_2 = Q \\ Q_2 = \frac{b}{a} Q_1 \end{cases} \Longrightarrow \begin{aligned} Q_1 &= \frac{a}{a+b} Q \\ Q_2 &= \frac{b}{a+b} Q \end{aligned}$$

Problem 2

For a very long conductor. the B field can be calculated according to the following formula (in Cylindrical Coordinate system)

$$B = \frac{\mu_0 I}{2\pi r} \hat{e}$$

a) At point (0. ±2m) the B field is:

$$B = \frac{\mu_0 i}{2\pi(2-0.005)}(-\hat{z}) + \frac{\mu_0 i}{2\pi(2+0.005)}\hat{z}$$

$$= 2\cdot10^{-7}\cdot 5.0\sin(50\pi t)\left(\frac{1}{2.005} - \frac{1}{1.995}\right)\hat{z}$$

$$= 10^{-6}\sin(50\pi t)\cdot(-0.0025)\hat{z}$$

$$= -2.5\ 10^{-9}\sin(50\pi t)\hat{z}$$

b) At center point, the B field is

$$B = 2\cdot\frac{\mu_0 i}{2\pi\cdot 0.005}\hat{z}$$

$$= 2\cdot 2\cdot10^{-7}\cdot 5.0\sin(50\pi t)\cdot 200$$

$$= 4.0\ 10^{-4}\sin(50\pi t)\hat{z}$$

Problem 3

(a) Mean value of the current is.

$$\langle i \rangle = \frac{1}{T}\int_0^T i\,dt$$

$$= \frac{1}{7}[5\cdot 10 + 2\cdot(-5)]$$

$$= \frac{1}{7}[50 - 10]$$

$$= \frac{40}{7} \approx 5.71 \text{ mA}$$

(b) Mean power is:

$$\langle p \rangle = \langle i^2 R \rangle = \langle i^2 \rangle R$$

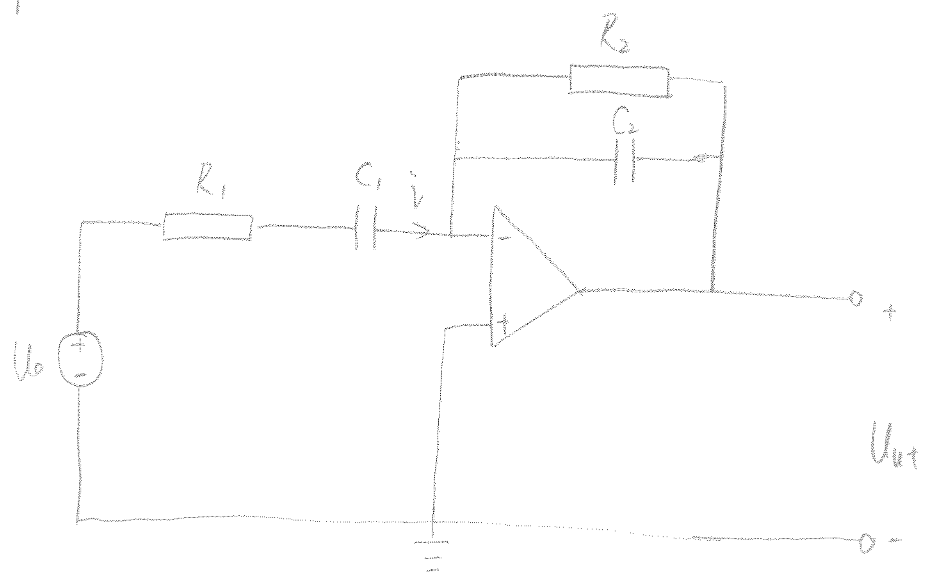
$$= \frac{1}{T}\int_0^T i^2\,dt \cdot R$$

$$= \frac{1}{7}[100\cdot 5 + 25\cdot 2]\cdot 50$$

$$= \frac{1}{7}[500 + 50]\cdot 50$$

$$= \frac{550}{7}\cdot 50 \approx 3928.6\,\mu W \approx 3.9 \text{ mW}$$

Problem 4



Assume the amplifier is ideal

According to the above figure we have

$$i\left(R_1 + \frac{1}{j\omega C_1}\right) = U_0 \Rightarrow i = \frac{U_0 \cdot j\omega C_1}{1+j\omega R_1 C_1}$$

$$i\left[\frac{1}{\frac{1}{R_2} + j\omega C_2}\right] + U_{ut} = 0 \Rightarrow U_{ut} = -\frac{R_2}{1+j\omega R_2 C_2} i$$

$$U_{ut} = -\frac{R_2}{1+j\omega R_2 C_2} \cdot \frac{j\omega C_1}{1+j\omega R_1 C_1} U_0$$

$$= -\frac{j\omega C_1 R_2}{(1+j\omega R_1 C_1)(1+j\omega R_2 C_2)} U_0$$

Problem 5

c) $g(x,y) = f(x,y) * h(x,y)$

$$= \sum_{a=-1}^{1} \sum_{b=-1}^{1} f(a,b)\, h(x-a, y-b)$$

$$g(0,0) = f(-1,-1)h(1,1) + f(-1,0)h(1,0) + f(-1,1)h(1,-1) +$$
$$f(0,-1)h(0,1) + f(0,0)h(0,0) + f(0,1)h(0,-1) +$$
$$f(1,-1)h(-1,1) + f(1,0)h(-1,0) + f(1,1)h(-1,-1)$$

$$= 4 \cdot 0 + 1 \cdot 1 + 4 \cdot 0 + 1 \cdot 1 + 6 \cdot (-2) + 1 \cdot 1 + 2 \cdot 0 + 1 \cdot 1 + 2 \cdot 0$$

$$= 0 + 1 + 1 - 12 + 1 + 1$$

$$= -12 + 4$$

$$= -8$$