Problem 1

a) $E = \frac{U}{d} = \frac{5V}{5mm} = \frac{5}{5 \cdot 10^{-3}} = 10^{3}\ V/m$

b) $C_{air} = \varepsilon_0 \frac{A_{air}}{d} = \varepsilon_0 \frac{\frac{A}{2}}{d} = \frac{\varepsilon_0 A}{2d}$

$C_{plastic} = \varepsilon_0 \varepsilon_r \frac{A_{plastic}}{d} = \frac{\varepsilon_0 \varepsilon_r A}{2d}$

$C_{total} = C_{air} + C_{plastic} = \frac{A}{2d} \cdot \varepsilon_0 (\varepsilon_r + 1) = \frac{3 \cdot 10^{-4}}{2 \cdot 5 \cdot 10^{-3}} \cdot 4 \cdot \frac{1}{36\pi} \cdot 10^{-9}$

$\simeq 1.06\ pF$

Problem 2 :

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\Phi_B = \int B \cdot ds$$

$$= \int_{r=5cm}^{r=13cm} \frac{\mu_0 I}{2\pi r} \cdot 10 \cdot 10^{-2} \cdot dr$$

$$= 0.1 \frac{\mu_0 I}{2\pi} \int_{r=5cm}^{r=13cm} \frac{1}{r} \cdot dr$$

$$= 0.1 \frac{4\pi \cdot 10^{-7} \cdot 10 \cos(20\pi t)}{2\pi} \ln \frac{13}{5}$$

$$= 2 \cdot 10^{-7} \cos(20\pi t) \ln \frac{13}{5}$$

$$\simeq 1.9 \cdot 10^{-7} \cos(20\pi t)$$

$$e = -\frac{d\Phi_B}{dt} = 1.9 \cdot 10^{-7} \cdot 20\pi \sin(20\pi t) \ V$$

$$\simeq 12 \cdot 10^{-6} \sin(20\pi t) \ V$$

$$\simeq 12 \sin(20\pi t) \ \mu V$$

Problem 3:

a)
$$I_e = \frac{U_e}{\sqrt{R^2 + (\frac{1}{wC})^2}}$$

$$U_e^C = I_e \cdot Z_c = \frac{U_e}{\sqrt{R^2 + (\frac{1}{wC})^2}} \cdot \frac{1}{wC} = 110V$$

$$wC \sqrt{R^2 + (\frac{1}{wC})^2} = 2$$

$$(wC)^2 [R^2 + (\frac{1}{wC})^2] = 4$$

$$R^2 (wC)^2 + 1 = 4$$

$$R^2 (wC)^2 = 3$$

$$R^2 = \frac{3}{(wC)^2} \Rightarrow R = \frac{\sqrt{3}}{wC}$$

$$= \frac{\sqrt{3}}{100\pi \cdot 5 \cdot 10^{-6}}$$

$$= \frac{\sqrt{3} \cdot 10^6}{500\pi}$$

$$\approx 1.1 \ k\Omega.$$

b)
$$I_e = 110 \cdot wC$$

$$= 110 \cdot 100\pi \cdot 5 \cdot 10^{-6}$$

$$= 1.1 \cdot 10^4 \pi \cdot 5 \cdot 10^{-6}$$

$$= 5.5 \cdot 10^{-2} \pi$$

$$\approx 17.2788 \cdot 10^{-2}$$

$$\approx 0.17 A$$

$$P_e = I_e^2 \cdot R \approx (0.17)^2 \cdot 1.1 \cdot 10^3$$

$$\approx 31.79 \ W$$

Problem 4

$$\bar{U}_{in} = 10e^{j100\pi t}$$

$$\bar{I}\left(R_1 + \frac{1}{j\omega C}\right) = \bar{U}_{in} \Rightarrow \bar{I} = \frac{\bar{U}_{in}}{R_1 + \frac{1}{j\omega C}}$$

$$\bar{I}\left(R_1 + R_2 + \frac{1}{j\omega C}\right) + \bar{U}_{ut} = \bar{U}_{in}$$

$$\Rightarrow \bar{I}R_2 + \bar{I}\left(R_1 + \frac{1}{j\omega C}\right) + \bar{U}_{ut} = \bar{U}_{in}$$

$$\Rightarrow \bar{I}R_2 + \bar{U}_{in} + \bar{U}_{ut} = \bar{U}_{in}$$

$$\Rightarrow \bar{U}_{ut} = -\bar{I}R_2 = -\frac{\bar{U}_{in}R_2}{R_1 + \frac{1}{j\omega C}} \simeq -\frac{10e^{j100\pi t} \cdot 50 \cdot 10^3}{5.93 \cdot 10^3 e^{-j0.5669}}$$

$$\simeq -84.3e^{j(100\pi t + 0.5669)}$$

$$U_{ut} = Im\{\bar{U}_{ut}\} = -84.3\sin[100\pi t + 0.5669]$$

$$(\bar{I} + \bar{i}_o)R_L = \bar{U}_{ut}$$

$$\Rightarrow \bar{I} + \bar{i}_o = \frac{\bar{U}_{ut}}{R_L}$$

$$\Rightarrow \bar{i}_o = \frac{\bar{U}_{ut}}{R_L} - \bar{I} = -\frac{\bar{U}_{in}R_2}{R_L\left(R_1 + \frac{1}{j\omega C}\right)} - \frac{\bar{U}_{in}}{R_1 + \frac{1}{j\omega C}}$$

$$= -\frac{\bar{U}_{in}(R_2 + R_L)}{R_L\left(R_1 + \frac{1}{j\omega C}\right)}$$

$$\simeq -\frac{10e^{j100\pi t} \cdot 55 \cdot 10^3}{5 \cdot 10^3 \cdot 5.93 \cdot 10^3 e^{-j0.5669}}$$

$$\simeq -0.0185e^{j(100\pi t + 0.5669)}$$

$$i_o = Im\{\bar{i}_o\} = -0.0185\sin[100\pi t + 0.5669]\ A$$

Problem 5

a)

$A \leftrightarrow 3$

$B \leftrightarrow 4$

$C \leftrightarrow 1$

$D \leftrightarrow 2$

sharp edges correspond to high frequency components.

Smooth parts of the picture are low frequency components

b)

$$g(x,y) = \Sigma f(a,b)\, h(x-a, y-b)$$

$$g(0,0) = f(-1,-1)\,h(1,1) + f(-1,0)\,h(1,0) + f(-1,1)\,h(1,-1)$$
$$+ f(0,-1)\,h(0,1) + f(0,0)\,h(0,0) + f(0,1)\,h(0,-1)$$
$$+ f(1,-1)\,h(-1,1) + f(1,0)\,h(-1,0) + f(1,1)\,h(-1,-1)$$

$$= 0 + 1\cdot 8 + 0 + 1\cdot 6 + (-4)\cdot 5 + 1\cdot 4 + 0 + 1\cdot 2 + 0$$

$$= 8 + 6 - 20 + 6$$

$$= 0$$