Problem 1

According to Gauss law

$$E = \frac{\Phi}{4\pi\varepsilon_0 r^2}$$

When $r < a$

$$\Phi = \int \rho_v \, dv$$
$$= \int_0^r \rho_v \cdot 4\pi r^2 dr$$
$$= \int_0^r \frac{\rho_0}{r} 4\pi r^2 dr$$
$$= \int_0^r 4\pi \rho_0 r \, dr$$
$$= 4\pi \rho_0 \frac{r^2}{2} \Big|_0^r$$
$$= 4\pi \rho_0 \cdot \frac{r^2}{2} = 2\pi \rho_0 r^2$$

$$E = \frac{\cancel{2}\pi \rho_0 r^2}{\cancel{4}\pi\varepsilon_0 r^2} = \frac{\rho_0}{2\varepsilon_0}$$

When $r \geq a$

$$\Phi = \int_0^a \frac{\rho_0}{r} 4\pi r^2 dr$$
$$= \int_0^a 4\pi \rho_0 r \, dr$$
$$= 2\pi \rho_0 r^2 \Big|_0^a$$
$$= 2\pi \rho_0 a^2$$

$$E = \frac{2\pi \rho_0 a^2}{4\pi\varepsilon_0 r^2}$$
$$= \frac{\rho_0 a^2}{2\varepsilon_0 r^2}$$

b) $$U_s = \int_{r=a}^{r=\infty} E \cdot dr$$
$$= \int_a^{\infty} \frac{\rho_0 a^2}{2\varepsilon_0 r^2} dr = \frac{\rho_0 a^2}{2\varepsilon_0} \left(-\frac{1}{r}\right) \Big|_a^{\infty} = \frac{\rho_0 a^2}{2\varepsilon_0 a} = \frac{\rho_0 a}{2\varepsilon_0}$$

Problem 2

$$L \geq r \qquad B = \frac{\mu_r \mu_0 I \cdot N}{L_{large}} = \frac{\mu_0 i N_{large}}{L_{large}}$$

$$\phi = B \cdot A_{small} = \frac{\mu_0 i N_{large} \cdot A_{small}}{L_{large}}$$

$$e = -N_{small} \frac{d\phi}{dt}$$

$$= -N_{small} \frac{d}{dt} \left[\mu_0 i N_{large} \frac{A_{small}}{L_{large}} \right]$$

$$= -N_{small} \mu_0 \cdot N_{large} \frac{A_{small}}{L_{large}} \cdot \frac{di}{dt}$$

$$= -\frac{\mu_0 N_{small} \cdot N_{large} \cdot A_{small}}{L_{large}} \frac{di}{dt}$$

$$= -\frac{4\pi \cdot 10^{-7} \cdot 200 \cdot 1500 \cdot 20 \cdot 10^{-4}}{0.5} \frac{d}{dt}(0.2 \sin 100\pi t)$$

$$= -\frac{8\pi \cdot 10^{-11} \cdot 3 \cdot 10^{5}}{0.5} \cdot 0.2 \cdot 100\pi \cos 100\pi t$$

$$= -\frac{8\pi \cdot 10^{-11} \cdot 3 \cdot 10^{5} \cdot 20\pi}{5 \cdot 10^{-1}} \cos 100\pi t$$

$$= -96\pi^2 \cdot 10^{-5} \cos 100\pi t$$

$$= -947.5 \cdot 10^{-5} \cos 100\pi t$$

$$= -9.475 \cdot 10^{-3} \cos 100\pi t$$

$$9.475 \cos 100\pi t \ \text{mV}$$

Problem 3

a) $U_e^C = \frac{220}{\sqrt{R^2+\left(\frac{1}{wC}\right)^2}} \cdot \frac{1}{wC}$

$= \frac{220}{wC \cdot \sqrt{R^2+\left(\frac{1}{wC}\right)^2}}$

$= \frac{220}{1000 \cdot 5.0 \cdot 10^{-6} \sqrt{200^2+\left(\frac{1}{5 \cdot 10^{-3}}\right)^2}}$

$= \frac{220}{200\sqrt{2} \cdot 5 \cdot 10^{-3}}$

$= \frac{220 \cdot 10^3}{200\sqrt{2} \cdot 5}$

$= 0.1556 \cdot 10^3$

$= 155.6\ V$

$U_e^R = \frac{220}{\sqrt{R^2+\left(\frac{1}{wC}\right)^2}} R$

$= \frac{220 R}{\sqrt{R^2+\left(\frac{1}{wC}\right)^2}}$

$= \frac{220 \cdot 200}{200\sqrt{2}}$

$= \frac{220}{\sqrt{2}}$

$= 155.6\ U$

b) $\hat{U} = \hat{I} \cdot \hat{Z}$

$\hat{Z} = \frac{\hat{U}}{\hat{I}}$ $\qquad \angle \hat{u} - \angle \hat{I} = \angle \hat{Z}$

$\hat{Z} = R + \frac{1}{jwC} = 200 - j200$

$tg\theta = \frac{-200}{200} = -1 \Rightarrow \theta = 225^\circ$

Problem 4

$$(R_1 + \frac{1}{j\omega C}) I = U_{in} \Rightarrow I = \frac{U_{in}}{R_1 + \frac{1}{j\omega C}}$$

$$I \left(\frac{1}{\frac{1}{R_2} + \frac{1}{j\omega L}} \right) + U_{ut} = 0$$

$$I \left(\frac{1}{\frac{j\omega L + R_2}{jR_2\omega L}} \right) = -U_{ut}$$

$$I \cdot \frac{jR_2\omega L}{R_2 + j\omega L} = -U_{ut}$$

$$\frac{U_{in}}{R_1 + \frac{1}{j\omega C}} \cdot \frac{jR_2\omega L}{R_2 + j\omega L} = -U_{ut}$$

$$\Rightarrow U_{ut} = - \frac{j\omega C}{1 + j\omega R_1 C} \cdot \frac{j\omega R_2 L}{R_2 + j\omega L} \cdot U_{in}$$

$$= - \frac{j\omega C}{1 + j\omega R_1 C} \cdot \frac{j\omega L}{1 + j(\frac{\omega L}{R_2})} U_{in}$$

Problem 5

$A \Longleftrightarrow 3$

$B \Longleftrightarrow 4$

$C \Longleftrightarrow 1$

$D \Longleftrightarrow 2$

$$g(x,y) = \sum f(a,b)\ h(x-a,\ y-b)$$

$$g(0,0) = f(-1,-1)\,h(1,1) + f(-1,0)\,h(1,0) + f(-1,1)\,h(1,-1)$$
$$+ f(0,-1)\,h(0,1) + f(0,0)\,h(0,0) + f(0,1)\,h(0,-1)$$
$$+ f(1,-1)\,h(-1,1) + f(1,0)\,h(-1,0) + f(1,1)\,h(-1,-1)$$

$$= 2\cdot 9 + 1\cdot 8 + 4\cdot 7 + 3\cdot 6 + 6\cdot 5 + 3\cdot 4 + 4\cdot 3$$
$$+ 1\cdot 2 + 2\cdot 1 = 130$$