Problem 1

a) According to Gauss law

$$\oint \vec{E} \cdot d\vec{s} = \frac{Q}{\varepsilon}$$

Cylindrical symmetry.

$$E_r \cdot 2\pi r L = \frac{Q}{\varepsilon}$$

E field inside the region between the two conductors

$$E_r \cdot 2\pi r L = \frac{\rho \cdot L}{\varepsilon} \Rightarrow E_r = \frac{\rho}{2\pi \varepsilon r}$$

E field outside the coaxial cable

$$E_r \cdot 2\pi r L = \frac{\rho \cdot L + (-\rho) \cdot L}{\varepsilon_0} = 0 \Longrightarrow E_r = 0$$

b)

$$U = \int_a^b E \cdot dr = \int_a^b \frac{\rho}{2\pi \varepsilon r} \cdot dr = \frac{\rho}{2\pi \varepsilon} \ln \frac{b}{a}$$

$$C = \frac{\rho}{U} = \frac{2\pi \varepsilon}{\ln b/a}$$

Problem 2

a) Ampere's current law

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 \mu_r N_1 I$$

Cylindrical symmetry:

$$B_\varphi \cdot 2\pi r = \mu_0 \mu_r N_1 I$$

$$B_\varphi = \frac{\mu_0 \mu_r N_1 I}{2\pi r}$$

b) $$\Phi_B = \int_S \vec{B}\cdot d\vec{s} = \int_a^b \int_0^h B_\varphi \cdot dz\, dr$$

$$= h \int_a^b B_\varphi\, dr$$

$$= \frac{\mu_0 \mu_r N_1 I \cdot h}{2\pi} \ln\frac{b}{a}$$

$$e = -N_2 \frac{d\Phi_B}{dt} = -\frac{\mu_0 \mu_r N_1 N_2 \cdot h}{2\pi} \ln\frac{b}{a} \frac{dI}{dt}$$

$$= -\frac{4\pi\cdot 10^{-7}\cdot 2500\cdot 2000\cdot 500\cdot 2\cdot 10^{-2}}{2\pi} \ln\frac{15}{10}\cdot 200\pi \cdot 2\cos(200\pi t)$$

$$= 5.095\cdot 10^{3} \cos(200\pi t)\ V$$

$$= 5.095 \cos(200\pi t)\ kV$$

Problem 3

a) $I_C = 0$; $I_L = I_R = \frac{U}{R} = \frac{10\,V}{50\,\Omega} = 0.2A$

b) $\bar{U} = 10 e^{j1000t}$

$$\bar{Z} = R + \frac{1}{j\omega C + \frac{1}{j\omega L}}$$

$$= R + \frac{1}{\frac{1-\omega^2 LC}{j\omega L}}$$

$$= R + \frac{j\omega L}{1-\omega^2 LC}$$

$$= \frac{R(1-\omega^2 LC) + j\omega L}{1-\omega^2 LC}$$

$$\bar{I}_R = \frac{\bar{U}}{\bar{Z}} = \frac{\bar{U}(1-\omega^2 LC)}{R(1-\omega^2 LC) + j\omega L} = (0.017 - j0.055) e^{j1000t}$$

$$\bar{I}_C = \bar{I}_R \cdot \frac{\bar{Z}_L}{\bar{Z}_C + \bar{Z}_L} = \bar{I}_R \cdot \frac{-\omega^2 LC}{1-\omega^2 LC} = (-0.011 + j0.037) e^{j1000t}$$

$$\bar{I}_L = \bar{I}_R \cdot \frac{\bar{Z}_C}{\bar{Z}_C + \bar{Z}_L} = \bar{I}_R \cdot \frac{1}{1-\omega^2 LC} = (0.028 - j0.092) e^{j1000t}.$$

Problem 4

$$\bar{I}\left(R_1 + \frac{1}{j\omega C_1}\right) = \bar{U}_{in} \Rightarrow \bar{I} = \frac{\bar{U}_{in}}{R_1 + \frac{1}{j\omega C_1}}$$

$$\bar{I}\left(\frac{1}{\frac{1}{R_2} + j\omega C_2}\right) + \bar{U}_{ut} = 0$$

$$\Rightarrow \bar{U}_{ut} = -\bar{I} \cdot \frac{1}{\frac{1}{R_2} + j\omega C_2}$$

$$= -\frac{\bar{U}_{in}}{R_1 + \frac{1}{j\omega C_1}} \cdot \frac{1}{\frac{1}{R_2} + j\omega C_2}$$

$$\Rightarrow \frac{\bar{U}_{ut}}{\bar{U}_{in}} = -\frac{j\omega C_1}{1 + j\omega R_1 C_1} \cdot \frac{R_2}{1 + j\omega R_2 C_2}$$

Problem 5

a)

Image 1 $\longleftrightarrow$ D

Image 2 $\longleftrightarrow$ E

Image 3 $\longleftrightarrow$ C

Image 4 $\longleftrightarrow$ B

Image 5 $\longleftrightarrow$ A

b) ramp filter, which is sort of high-pass filter.

The reason is to make the reconstruction less blurred,

and to enhance the fine structures in the image.