Solution 2019-08-26

Problem 1

a) $\oint_S \vec{E}\cdot d\vec{s} = \frac{Q}{\varepsilon_0}$

$\Rightarrow 4\pi R^2 E_r = \frac{Q}{\varepsilon_0}$

$\Rightarrow E_r = \frac{Q}{4\pi\varepsilon_0 R^2}$

$R<a$

$Q = \int_V \rho_v dv$

$= \int_0^R \frac{\rho_0}{r}\cdot 4\pi r^2 dr$

$= \int_0^R 4\pi\rho_0 r dr$

$= 4\pi\rho_0 \cdot \frac{r^2}{2}\Big|_0^R = 2\pi\rho_0 R^2$

$E_r = \frac{\rho_0}{2\varepsilon_0}$

$R \geq a$

$Q = \int_V \rho_v dv$

$= \int_0^a 4\pi\rho_0 r dr$

$= 2\pi\rho_0 a^2$

$E_r = \frac{\rho_0 a^2}{2\varepsilon_0 R^2}$

b) $V = \int_a^\infty E_r dR = \int_a^\infty \frac{\rho_0 a^2}{2\varepsilon_0 R^2} dR$

$= -\frac{\rho_0 a^2}{2\varepsilon_0 R}\Big|_a^\infty$

$= \frac{\rho_0 a}{2\varepsilon_0}$

Problem 2

a) Ampere's current law

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 \mu_r N_1 I$$

Cylindrical symmetry:

$$B_\theta \cdot 2\pi r = \mu_0 \mu_r N_1 I$$

$$B_\theta = \frac{\mu_0 \mu_r N_1 I}{2\pi r}$$

b) $$\Phi_B = \int_S \vec{B} \cdot d\vec{s} = \int_a^b \int_0^h B_\theta \cdot dz\, dr = h \int_a^b B_\theta\, dr$$

$$= \frac{\mu_0 \mu_r N_1 I h}{2\pi} \ln \frac{b}{a}$$

$$e = -N_2 \frac{d\Phi_B}{dt} = -\frac{\mu_0 \mu_r N_1 N_2 h}{2\pi} \ln \frac{b}{a} \frac{dI}{dt}.$$

$$= -5.095 \cos(200\pi t)\ \text{kV}$$

Problem 3

a)

$$\begin{cases} (\bar{I}_c + \bar{I}_R) j\omega L + \bar{I}_R R = \bar{U} & ① \\ \bar{I}_c \frac{1}{j\omega C} = \bar{I}_R R & ② \end{cases}$$

From ②

$$\bar{I}_c = j\omega RC \bar{I}_R$$

①
$$\Rightarrow (\bar{I}_R + j\omega RC \bar{I}_R) j\omega L + \bar{I}_R R = \bar{U}$$

$$\bar{I}_R = \frac{\bar{U}}{R + (1 + j\omega RC) j\omega L}$$

$$\bar{U}_c = \bar{U}_R = \bar{I}_R R = \frac{R\bar{U}}{R + (1+j\omega RC) j\omega L} = (0.1376 - j0.4587) 10 e^{j1000t}$$

$$= 0.4789 e^{-j1.2793} \cdot 10 e^{j1000t}$$

$$= 4.789 e^{j(1000t - 1.2793)}$$

$$\bar{U}_L = \bar{U} - \bar{U}_c = \bar{U} \left[1 - \frac{R}{R + (1+j\omega RC) j\omega L}\right] = (0.8624 + j0.4587) \cdot 10 e^{j1000t}$$

$$= 0.9768 \cdot e^{j0.4889} \cdot 10 e^{j1000t}$$

$$= 9.768 e^{j(1000t + 0.4889)}$$

b)

$$P = \frac{1}{2} \frac{\hat{U}_R^2}{R} = \frac{1}{2} \frac{\hat{U}_c^2}{R} = \frac{1}{2} \cdot \frac{(4.789)^2}{50} = 0.2293 W$$

Problem 4.

$$\begin{cases} \bar{I}R = \bar{U}_{in} & ① \\ \bar{I}\frac{1}{j\omega C} + \bar{U}_{ut} = 0 & ② \end{cases}$$

From ①

$$\bar{I} = \bar{U}_{in}/R$$

Insert ① into ②

$$\frac{\bar{U}_{in}}{R}\frac{1}{j\omega C} = -\bar{U}_{ut}$$

$$H(j\omega) = \frac{\bar{U}_{ut}}{\bar{U}_{in}} = -\frac{1}{j\omega RC}.$$

Problem 5

a) Histogram A is used.

From image 1 to image 2, dark parts become darker. bright parts become brighter. Histogram A gives the corresponding mapping.

b)

0	0	0
1	2	1
0	2	0

c) In microwave tomography, microwave signals are transmitted into the object under detection and the scattered fields are measured at numerous different locations. By utilizing the received signal, the dielectric properties of the object is reconstructed.