Problem 1

(a). According to Gauss law

$$\oint \vec{E} \cdot \vec{ds} = \frac{Q}{\varepsilon_0}$$

$$E_R = \frac{Q}{4\pi\varepsilon_0 r^2}$$

For $r \leq a$

$$Q = \int_0^r \rho_v dv = \int_0^r \frac{\rho_0}{r} \cdot 4\pi r^2 dr = 4\pi\rho_0 \left(\frac{r^2}{2}\right) = 2\pi\rho_0 r^2$$

For $r \geq a$ $\quad Q = \int_0^a \rho_v dv = 2\pi\rho_0 a^2$

$$E_R = \frac{2\pi\rho_0 r^2}{4\pi\varepsilon_0 r^2} = \frac{\rho_0}{2\varepsilon_0} \qquad r \leq a$$

$$E_R = \frac{2\pi\rho_0 a^2}{4\pi\varepsilon_0 r^2} = \frac{\rho_0 a^2}{2\varepsilon_0 r^2} \qquad r \geq a$$

b)
$$V = \int_a^{\infty} E_R dr = \int_a^{\infty} \frac{\rho_0 a^2}{2\varepsilon_0 r^2} dr = \frac{\rho_0 a^2}{2\varepsilon_0}\left[-\frac{1}{r}\right]_a^{\infty}$$

$$= \frac{\rho_0 a}{2\varepsilon_0} \text{ V}$$

Problem 2

B field produced by the large coil is:

$$B_L = \frac{\mu_0 i N_L}{l_L}$$

The magnetic flux going through the small coil is:

$$\Phi_S = A_S B_L$$

According to Faraday's induction law:

$$e = -N_S \frac{d\Phi_S}{dt} = -N_S \frac{\mu_0 N_L A_S}{l_L} \frac{di}{dt}$$

$$= -1600 \cdot 200 \cdot 4\pi \cdot 10^{-7} \cdot 2 \cdot 10^{-4} \cdot \frac{1}{0.4} \frac{d}{dt}(0.2 \sin 100\pi t)$$

$$= -1600 \cdot 200 \cdot 4\pi \cdot 10^{-7} \cdot 2 \cdot 10^{-4} \cdot 0.2 \cdot 100\pi \cos 100\pi t$$

$$\approx -0.0126 \cos 100\pi t \text{ V}$$

Problem 3.

a) $I_C = 0 \quad I_R = \frac{220V}{50\Omega} = 4.4A$

b) $P = \frac{1}{2}\frac{\hat{U}_R^2}{R} = \frac{1}{2}\frac{(220\sqrt{2})^2}{50} = 968\,W$

Problem 4

$$\begin{cases} \bar{I}(R_1 + \frac{1}{j\omega C} + R_2) + \bar{U}_{ut} - \bar{U}_{in} = 0 & ① \\ \bar{I}(R_1 + \frac{1}{j\omega C}) - \bar{U}_{in} = 0 & ② \end{cases}$$

From ② $\bar{I} = \frac{\bar{U}_{in}}{R_1 + \frac{1}{j\omega C}}$

From ①, ② $\bar{I}R_2 + \bar{U}_{ut} = 0 \Rightarrow \bar{U}_{ut} = -\bar{I}R_2 = -\frac{\bar{U}_{in}}{R_1 + \frac{1}{j\omega C}}R_2$

$= 84.3564\, e^{j(100\pi t + 0.5669)}$

$\bar{U}_{ut} = (\bar{i}_0 + \bar{I})R_L \Rightarrow \bar{i}_0 = \frac{\bar{U}_{ut}}{R_L} - \bar{I}$

$= \frac{-\bar{U}_{in}R_2}{R_1 + \frac{1}{j\omega C}} - \frac{\bar{U}_{in}}{R_1 + \frac{1}{j\omega C}}$

$= 0.0186\, e^{j(100\pi t + 0.5669)}$

Problem 5

a)

A: 1×5

B: 3×3

C: 1×9

D: 5×5

E: 9×9

b)

$$g(x,y) = f(x,y) * h(x,y) = \sum_{-M}^{M} \sum_{-N}^{N} f(a,b)\, h(x-a, y-b)$$

$g(0,0) = 20$

$g(-1,-1) = 10$

$g(-1,0) = 12$

According to symmetry:

$$g(x,y) = \begin{bmatrix} 10 & 12 & 10 \\ 12 & 20 & 12 \\ 10 & 12 & 10 \end{bmatrix}$$