Experimental design (KKR031, KBT120)

Tuesday 23/8 2011 - 8:30-13:30 V

Jan Rodmar will be available at ext 3024 and will visit the examination room at ca 10:30.

The examination results will be available for review at the earliest Friday 9/9 2011.

Time for examination = 5 h

Examination aids:

Textbook (Douglas C. Montgomery: Design and Analysis of Experiments) with notes. No calculation examples (in book or on paper) are allowed as aid. All type of calculators are allowed. Standard Math. Tables, TEFYMA table, Beta Mathematics Handbook or Handbook of Chemistry and Physics are accepted.

Problem 1 (5 credits)

An article in Solid State Technology describes an experiment to determine the effect of C_2F_6 flow rate on the uniformity of the etch on a silicon wafer used in integrated circuit manufacturing. Data for two flow rates are as follows:

C_2F_6	Uniformity observation										
Flow rate	1	2	3	4	5	6					
125	2.7	4.6	2.6	3.0	3.2	3.8					
200	4.6	3.4	2.9	3.5	4.1	5.1					

A: Does the C₂F₆ flow rate affect average etch uniformity?

B: Does the C_2F_6 flow rate affect the wafer-to-wafer variability in etch uniformity?

Use $\alpha = 0.05$.

Problem 2 (7 credits)

A consumer product-testing organization has compared the annual power consumption for five different brands of dehumidifier. Because power consumption depends on the prevailing humidity level, it was decided to monitor each brand at four different humidity levels. Within each level, brands were randomly assigned to the five selected locations. The resulting amount of power consumption (kWh/year) appears in the following table.

Dranda	Humidity level								
Dialius	1	2	3	4					
1	685	792	838	875					
2	722	806	893	953					
3	733	802	880	941					
4	811	888	952	1005					
5	828	920	978	1023					

A: Evaluate if the brands have a significant (95%) effect on the power consumption.

B: Calculate even if there is a significant difference between brands:

B1: 2 and 3 B2: 4 and 5

C: Comment the results in A and B.

Problem 3 (8 credits)

It is of interest to investigate the effects of five factors on the colour of a chemical product. The factors are A = solvent, B = catalyst, C = temperature, D = reactant purity and E = pH. Experiments with 16 runs have been performed and the results are as follows:

e =	-0.63	<i>d</i> =	6.79
a =	2.51	ade =	5.47
b =	-2.68	bde =	3.45
abe =	1.66	abd =	5.68
<i>c</i> =	2.06	cde =	5.22
ace =	1.22	acd =	4.38
bce =	-2.09	bcd =	4.30
abc =	1.93	abcde =	4.05

- A: What kind of a fractional factorial design is used and what is the alias structure? Four-factor interactions can be omitted.
- B: Calculate the main effects and the two-factor interaction effects AB, AC, AD and CD and examine their significance ($\alpha = 0.05$). All other factor effects can be neglected.
- C: Calculate the residuals and plot them versus the fitted values. Comment on the plot.

 $SS(corr) = \sum (y_i - \overline{y})^2 = 114.7$

Problem 4 (6 credits)

One wishes to develop a new substrate for production of a single cell protein consisting of N-source (N), Carbon-source (C) and a Potassium content (K). One has earlier shown that

 $\begin{array}{l} N \; > 10 \; \% \\ C \; > 60 \; \% \\ K \; > 2 \; \% \end{array}$

In order to find a good optimum one wants to fit a polynomial with both linear, interaction and quadratic terms.

A: Suggest a suitable experimental design and calculate the composition of the substrate in the suggested experiments.

There must be at least four degrees of freedom for determination of the experimental error.

B: Calculate even the first row in the calculation matrix for the suggested design.

Problem 5 (8 credits)

The model
$$\hat{y} = b_0 + \sum_{i=1}^{3} b_i x_i + \sum_{i=1}^{3} \sum_{j>i}^{3} b_{ij} x_i x_j$$
 has been fitted to the following experimental

data

Y	12.48	44.14	26.32	23.71	19.93	24.67	24.16	17.50	25.49	24.45	31.15	12.75	17.66	16.15	41.80	22.20
X_1	-1.00	-1.00	-1.00	1.00	1.00	0	1.00	1.00	1.00	1.00	-1.00	1.00	1.00	1.00	-1.00	0
X_2	-1.00	1.00	-1.00	1.00	0	0	-1.00	1.00	0	1.00	1.00	-1.00	1.00	1.00	1.00	0
X ₃	-1.00	1.00	1.00	-1.00	1.00	0	-1.00	1.00	0	-1.00	0	1.00	1.00	1.00	1.00	0

with the result:

 $\mathbf{b}^{\mathrm{T}} = [23.29 \ -2.51 \ 3.46 \ 2.24 \ -3.03 \ -6.51 \ 1.22]$

The diagonal elements in $(\mathbf{X}^{T}\mathbf{X})^{-1}$ have been calculated to :

 $[0.09 \ 0.10 \ 0.12 \ 0.12 \ 0.11 \ 0.15 \ 0.13]$

Useful sums of squares:

$$SS_{T} = \sum (y_i - \overline{y})^2 = 1205.07$$
 $SS_{E1} = \sum (y_i - \hat{y}_i)^2 = 54.97$

A: Decide if the model is significant.

B: Calculate 95 % confidence interval for parameter b₁ (-2.51).

C: Examine if the model has systematic errors.

When even quadratic terms were added to the model the residual sum of squares decreased to

$$SS_{E2} = \sum (y_i - \hat{y}_i)^2 = 10.01$$

D: Calculate if the added parameters significantly improved the model.

Use $\alpha = 0.05$

Problem 6 (6 credits)

A: Describe how one can find the optimum in an experimental region with

- 1. Augmented experimental design and canonical analysis
- 2. Steepest ascent (Gradient search)
- 3. EVOP
- B: Also state under which conditions the different methods are suitable.