Design and Analysis of Experiments/Försöksplanering

(KBT120, KKR031)

Thursday 17/1 2013   8:30-13:30   M

Jan Rodmar will be available at ext 3030 and will visit the examination room ca 10:30.

The examination results will be available for review 1/2 12:45 – 13:15 KRT sem. room.

Time for examination = 5 h

Examination aids:

Textbook (Douglas C. Montgomery: Design and Analysis of Experiments) with notes. No calculation examples (in book or on paper) are allowed as aid.

All type of calculators are allowed.

Standard Math. Tables, TEFYMA table, Beta Mathematics Handbook or Handbook of Chemistry and Physics and Language dictionaries are accepted.

Problem 1 (8 credits)

In semiconductor manufacturing, wet chemical etching is often used to remove silicon from the backs of wafers prior to metallization. The etch rate is an important characteristic of this process. Two different etching solutions are being evaluated. Eight randomly selected wafers have been etched in each solution and the observed etch rates (in $\times 10^{-3}$ cm/min) are shown below:

Solution 1	9.9	9.4	10.0	10.3	10.6	10.3	9.3	9.8
Solution 2	10.2	10.0	10.7	10.5	10.6	10.2	10.4	10.3

A: Test the hypotheses that the variances for solution 1 and 2 are equal.

B: Do the data indicate that the claim that both solutions have the same mean etch rate. is valid?

Use the significance level: $\alpha = 0.05$.

Problem 2 (10 credits)

A process engineer is testing the yield of a product manufactured on three machines. Each machine has three stations on which the product is formed. An experiment is conducted in which two observations on yield are taken from each station. The runs are made in random order, and the results follow.

Machine	**I**			**II**			**III**		
Station	1	2	3	1	2	3	1	2	3
Yield	35	34	37	31	33	32	31	33	32
	32	36	38	34	34	35	32	31	30

A: Analyze this experiment assuming that the two factors are fixed. Comment the results.

B: Estimate if there is any significant difference between the three machines. Is any machine to prefer?

Use $\alpha = 0.05$

Problem 3 (12 credits)

An article in the *Journal of Quality Technology* (Vol.17, 1985, pp.198-206) describes the use of fractional factorial design to investigate the effect of five factors on the free height of leaf springs used in an automotive application. The factors are A = furnace temperature, B = heating time, C = transfer time, D = hold down time, and E = quench oil temperature. The design and the observed free height on each run are shown in the table:

Design					Free Height
A	**B**	**C**	**D**	**E**	
–	–	–	–	–	7.81
+	–	–	+	–	7.88
–	+	–	+	–	7.50
+	+	–	–	–	7.75
–	–	+	+	–	7.88
+	–	+	–	–	8.06
–	+	+	–	–	7.44
+	+	+	+	–	7.69
–	–	–	–	+	7.12
+	–	–	+	+	7.44
–	+	–	+	+	7.50
+	+	–	–	+	7.56
–	–	+	+	+	7.44
+	–	+	–	+	7.62
–	+	+	–	+	7.25
+	+	+	+	+	7.59

A: Indentify the defining relation and write out the alias structure for this design. What is the resolution of this design?

B: Calculate the main factor effects and two-factor interaction effects AD, AE, BE and DE and examine which factors influence the free height. All other factors can be neglected. α = 0.05

C: Analyze the residuals and comment on your findings.

D: Is this the best possible design for five factors in 16 runs?

Problem 4 (10 credits)

A drink bottler is interested in obtaining a more uniform fill heights in the bottles produced by his manufacturing process.The process engineer can control three variables during the filling process: percent carbonation (x_1), operating pressure in the filler (x_2) and bottles produced per minute or line speed (x_3).
He carried out experiment according to a factorial design in these three variables, and observed the average deviation from the target fill height. A regression model was fitted to the data.

In the larger production plant the percentage of carbonation is a noise variable so the response model can be written in terms of coded variables as

$$y=\beta_0 + \gamma_1 z_1 + \beta_2 x_2 + \beta_3 x_3 + \delta_{21} x_2 z_1 + \varepsilon$$

with process variables: x_2 = pressure
x_3 = line speed
noise variable: z_1 = percent carbonation

response: y = fill height deviation

The parameters have been estimated to:

$\beta_0 = 1.00$ $\beta_2 = 1.13$ $\beta_3 = 0.88$ $\gamma_1 = 1.50$ $\delta_{21} = 0.38$

The variance in the response is $s^2 = 0.66$ (the residual mean square) and the variance in the noise variable is assumed to be $s_z^2 = 1$.

Find a set of conditions that result in mean fill height deviation as close to zero as possible. with minimum transmitted variance.

Problem 5 (12 credits)

In an automated chemical coating process, the speed with which objects on conveyor belt are passed through a chemical spray (belt speed = x_2), the amount of chemical sprayed (spray volume = x_1) , and the brand of chemical used (brand = x_3) are factors that may affect the uniformity of the coating applied. A replicated 2^3-experiment was conducted in an effort to increase the coating uniformity. In the following table, higher values of the response variable are associated with higher surface uniformity.

x_1	− 1	1	− 1	1	− 1	1	− 1	1
x_2	− 1	− 1	1	1	− 1	− 1	1	1
x_3	− 1	− 1	− 1	− 1	1	1	1	1
Response 1	40	25	30	50	45	25	30	52
Response 2	36	28	32	48	43	30	29	49

A: Fit the model

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{23} x_2 x_3 + \beta_{123} x_1 x_2 x_3 + \varepsilon$$

to the data and examine which factors appear to have an effect on surface uniformity.

B: Decide if the model is significant .

C: Estimate the predicted value and the prediction interval in the point x=[1 1 1].

Use $\alpha = 0.05$

Problem 6 (8 credits)

The yield (y) of a process depends on three variables (**x**) and has been examined according to the following design:

x_1	x_2	x_3
0	$\sqrt{2}$	-1
$-\sqrt{2}$	0	1
0	$-\sqrt{2}$	-1
$\sqrt{2}$	0	1

The purpose is to estimate the parameters in the model

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$$

One suspects that the true relationship is somewhat nonlinear and this property can be described by adding the squared terms

$$\beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{33} x_3^2 \; .$$

If this is the case how will this affect the least square estimate of β_0, β_1, β_2 and β_3 with b_0 , b_1 , b_2 and b_3 ?

What are then the expected values of b_0 , b_1, b_2 and b_3 ?