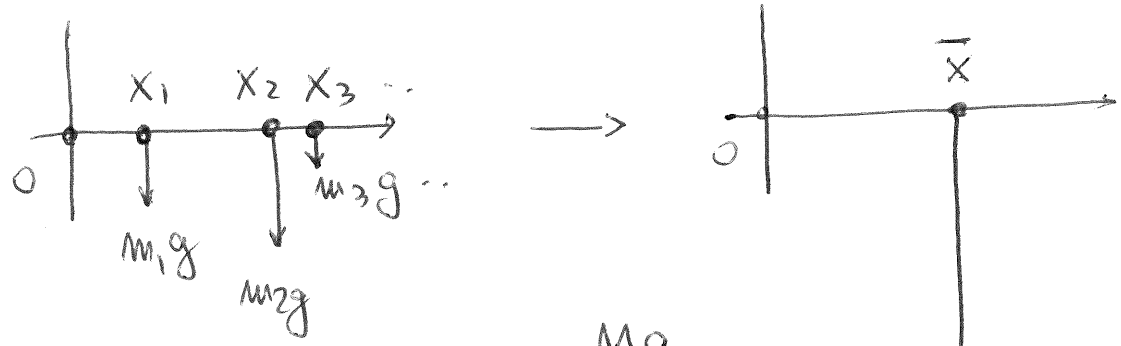
MASSCENTRUM:



$$\begin{cases} m_1 g + m_2 g + \cdots = Mg \\ m_1 g x_1 + m_2 g x_2 + \cdots = Mg\bar{x} \end{cases}$$

$$\Rightarrow M = \sum_i m_i$$

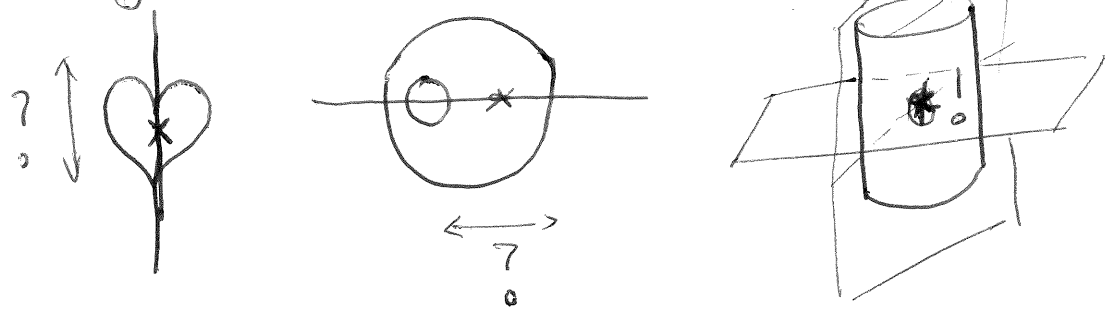
$$\bar{x} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

(Om g varierar, t.ex. månens massc. med avseende på jordens tyngkraft, då är massc. $\neq$ tyngpkt.)

I 3D:

$$\bar{\mathbf{r}} = \frac{\sum_i m_i \mathbf{r}_i}{\sum_i m_i}$$

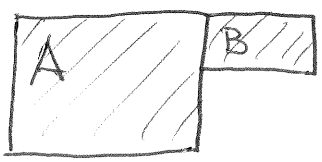
* Om kroppen har ett symmetry, ligger masscentrum i symmetryplan / symmetryaxeln.



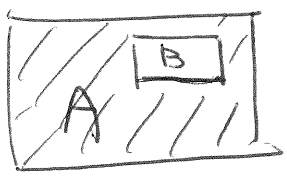
* Kontinuerlig massfördelning:

$$\bar{r} = \frac{\sum r_i m_i}{M} \rightarrow \frac{\int r\,dm}{M} = \frac{\int r \rho\,dV}{M}.$$

* UPPDELNING (Sammansatta kroppar)



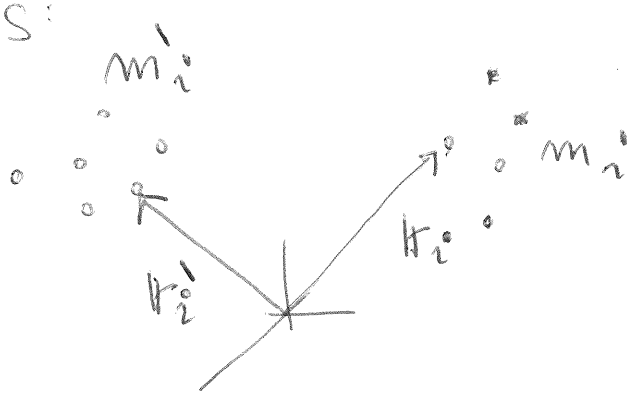
$$\bar{X} = \frac{M_A \bar{X}_A + M_B \bar{X}_B}{M_A + M_B}$$



$$\bar{X} = \frac{M_A \bar{X}_A - M_B \bar{X}_B}{M_A - M_B}$$

Notera att vid ρ KONST $M \propto$ Volym (eller ytan i 2D!).

Bevis:

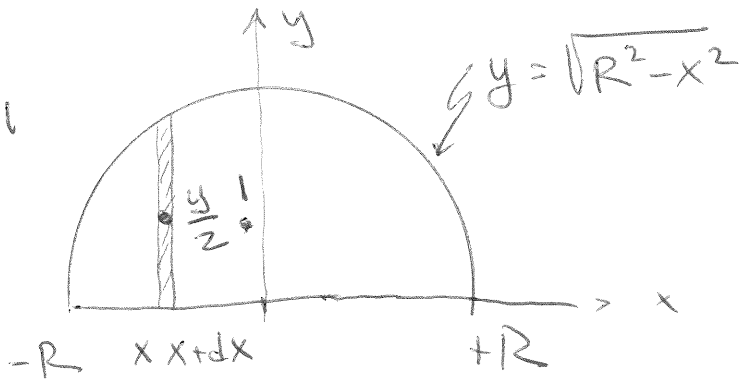


$$\vec{r}' = \frac{\sum m_i' \vec{r}_i'}{\sum m_i'} \qquad \vec{r} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

$$\vec{R} = \frac{\sum m_i \vec{r}_i + \sum m_i' \vec{r}_i'}{\sum m_i + \sum m_i'} =$$

$$= \frac{(\sum m_i)\vec{r} + (\sum m_i')\vec{r}'}{\sum m_i + \sum m_i'} = \frac{M\vec{r} + M'\vec{r}'}{M + M'}$$

Ex 1



ytdensitet $\sigma = M / \frac{1}{2}\pi R^2 = \frac{2M}{\pi R^2}$

$dm = \sigma \cdot \sqrt{R^2-x^2}\, dx$.

$$\bar{y} = \frac{\int \frac{y}{2} dm}{M} = \frac{\sigma}{M} \int_{-R}^{+R} \frac{1}{2}(R^2-x^2)\, dx =$$

$$= \frac{1}{\pi R^2} \cdot R^3 \cdot \int_{-1}^{1} (1-z^2)\, dz = \frac{R}{\pi} \cdot \left(2 - \frac{2}{3}\right) =$$

$$= \frac{4R}{3\pi}$$