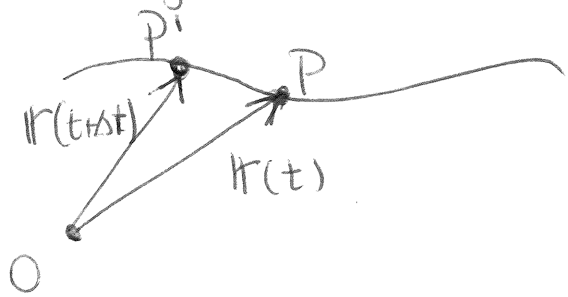
KROKLINJIG RÖRELSE.



$$\mathbf{v}(t) = \lim_{\Delta t \to 0} \frac{\mathbf{r}(t+\Delta t) - \mathbf{r}(t)}{\Delta t} = \frac{d\,\mathbf{r}(t)}{dt} = \dot{\mathbf{r}}(t)$$

$$\mathbf{a}(t) = \dot{\mathbf{v}}(t) = \ddot{\mathbf{r}}(t)$$

Detta är alltid sant. Men vad menas med $\frac{d\mathbf{r}}{dt}$?

Inför en <u>inertialsystem</u> med <u>Cartesiska koordinater</u>:

$$\mathbf{r} = x\,\mathbf{e}_x + y\,\mathbf{e}_y + z\,\mathbf{e}_z .$$

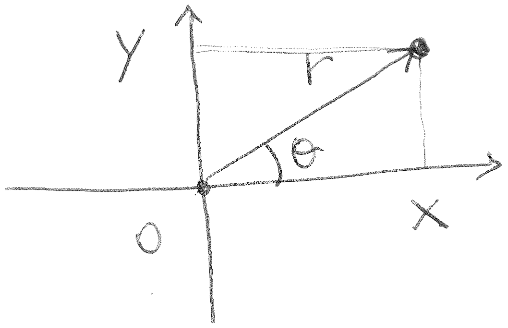
BARA i detta fall: $\frac{d\,\mathbf{e}_x}{dt} = 0$ o.s.v.

$$\mathbf{v} = \dot{x}\,\mathbf{e}_x + \dot{y}\,\mathbf{e}_y + \dot{z}\,\mathbf{e}_z$$

$$\Rightarrow \mathbf{a} = \ddot{x}\,\mathbf{e}_x + \ddot{y}\,\mathbf{e}_y + \ddot{z}\,\mathbf{e}_z .$$

OBS! DENNA DELEN FINNS INTE MED I BOKEN

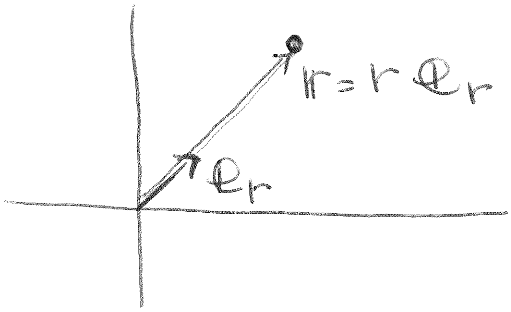
DÄREMOT: i en polärt koordinatsyst. (2D för enkelhetens skull).



$$x = r\cos\theta$$
$$y = r\sin\theta .$$

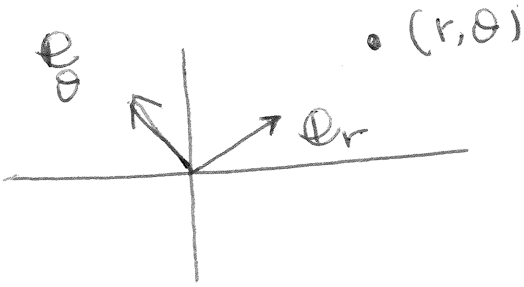
$$\mathbf{r} = x\,\mathbf{e}_x + y\,\mathbf{e}_y = r\cos\theta\,\mathbf{e}_x + r\sin\theta\,\mathbf{e}_y$$

$$\Rightarrow \quad \mathbf{e}_r = \frac{\mathbf{r}}{r} = \cos\theta\,\mathbf{e}_x + \sin\theta\,\mathbf{e}_y$$



Inför en annan enhetsvektor

$\mathbf{e}_\theta \perp \mathbf{e}_r$: $\quad \mathbf{e}_\theta = -\sin\theta\,\mathbf{e}_x + \cos\theta\,\mathbf{e}_y$



Observera att:

$$\dot{\mathbf{e}}_r = \frac{d}{dt}\left(\cos\theta\,\mathbf{e}_x + \sin\theta\,\mathbf{e}_y\right) =$$

$$= \frac{d\cos\theta}{dt}\mathbf{e}_x + \frac{d\sin\theta}{dt}\mathbf{e}_y =$$

$$= -\dot{\theta}\sin\theta\,\mathbf{e}_x + \dot{\theta}\cos\theta\,\mathbf{e}_y = \dot{\theta}\,\mathbf{e}_\theta \quad !$$

På samma sätt visar man att:

$$\dot{\mathbf{e}}_\theta = -\dot{\theta}\,\mathbf{e}_r .$$

Därför:

$$\mathbf{v} = \dot{\mathbf{r}} = \frac{d}{dt}(r\mathbf{e}_r) = \dot{r}\mathbf{e}_r + r\dot{\mathbf{e}}_r =$$

$$= \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta \equiv v_r\mathbf{e}_r + v_\theta\mathbf{e}_\theta .$$

$$\mathbf{a} = \dot{\mathbf{v}} = \frac{d}{dt}\left(\dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta\right) =$$

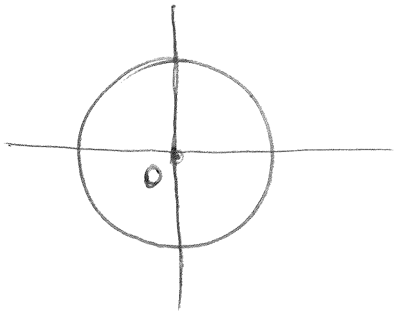
$$= \ddot{r}\mathbf{e}_r + \dot{r}\dot{\mathbf{e}}_r + \dot{r}\dot{\theta}\mathbf{e}_\theta + r\ddot{\theta}\mathbf{e}_\theta + r\dot{\theta}\dot{\mathbf{e}}_\theta =$$

$$= \ddot{r}\mathbf{e}_r + \dot{r}\dot{\theta}\mathbf{e}_\theta + \dot{r}\dot{\theta}\mathbf{e}_\theta + r\ddot{\theta}\mathbf{e}_\theta - r\dot{\theta}^2\mathbf{e}_r =$$

$$= (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta \equiv a_r\mathbf{e}_r + a_\theta\mathbf{e}_\theta$$

* För CIRKELRÖRELSE, $\underline{r = \text{konst.}}$

$(\dot{r} = \ddot{r} = 0)$.



$$\mathbf{v} = r\dot{\theta}\,\mathbf{e}_\theta$$

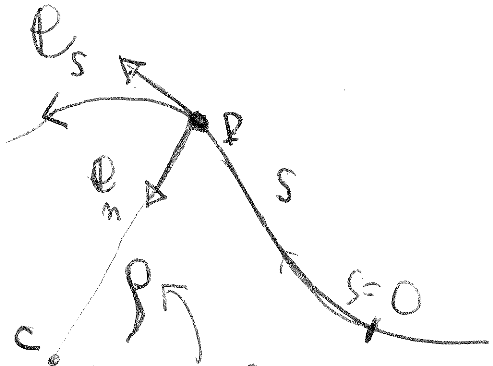
$$\dot{\theta} \equiv \omega \qquad v = r\dot{\theta} = r\omega$$

$$\mathbf{a} = \underbrace{-r\dot{\theta}^2\,\mathbf{e}_r}_{\substack{\text{centripetal}\\ \text{acceleration}\\ \text{(pekar mot O)}}} + \underbrace{r\ddot{\theta}\,\mathbf{e}_\theta}_{\substack{\text{Tangential}\\ \text{acceleration.}}}$$

$$r\dot{\theta}^2 = r\omega^2 = \frac{v^2}{r}$$

$$r\ddot{\theta} = r\dot{\omega} = r\alpha = \dot{v}$$

NATURLIGA RIKTNINGAR i ett plan



KURVANS KRÖKNINGSRADIE i P

$$\mathbf{v} = v\,\mathbf{e}_s$$

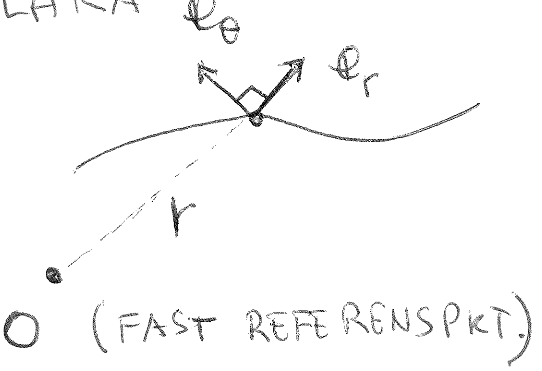
$$\mathbf{a} = \dot{v}\,\mathbf{e}_s + \frac{v^2}{\rho}\,\mathbf{e}_n$$

(ofta skriver man $\mathbf{e}_t$ istället för $\mathbf{e}_s$)

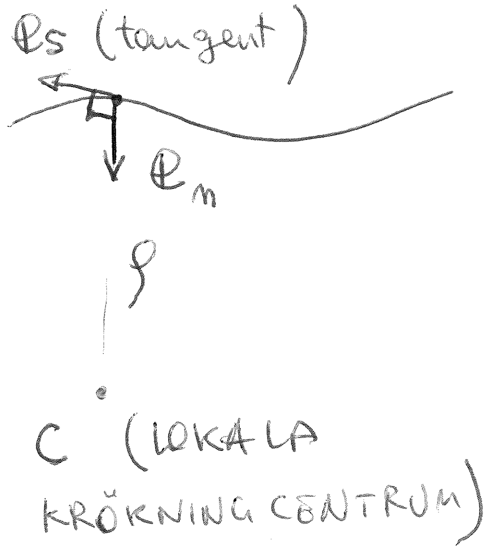
$$v \equiv \dot{s}$$

OBSERVERA SKILLNADEN !

POLÄRA



NATURLIGA



$$\dot{r}\,e_r + r\dot{\theta}\,e_\theta = \mathbb{V} = v\,e_s$$

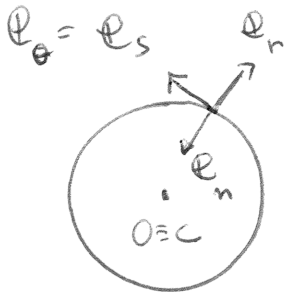
$$(\ddot{r} - r\dot{\theta}^2)\,e_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,e_\theta = a = \dot{v}\,e_s + \frac{v^2}{\rho}\,e_n.$$

CIRKELRÖRELSE

$r = \rho$ KONSTANT, $O \equiv C$ ALLTID

$e_n = -e_r$, $e_\theta = e_s$

$v = r\dot{\theta}$



$\omega = \dot{\theta}$ VINKELHASTIGHET (AGULAR VELOCITY)

$\alpha = \dot{\omega}$ VINKELACCELERATION (ANGULAR ACCELERATION)