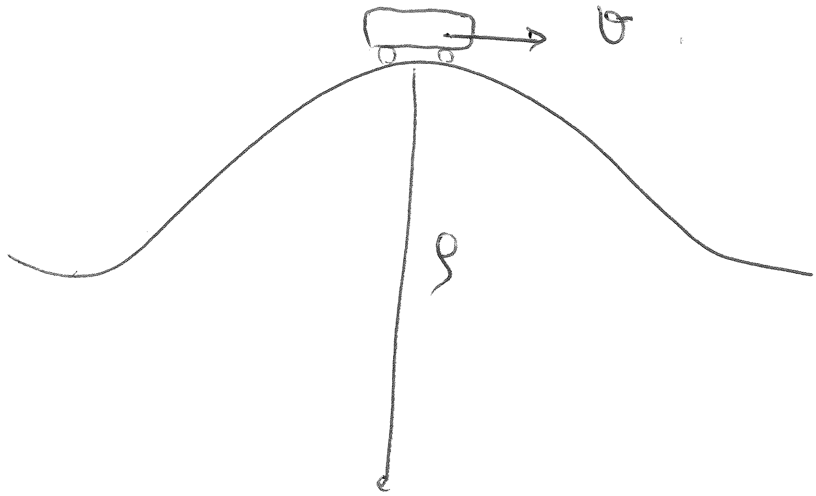
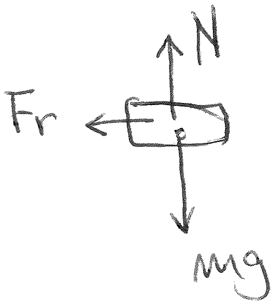
EXEMPEL



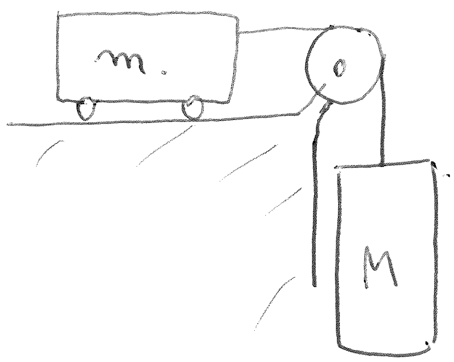
FRILÄGG:



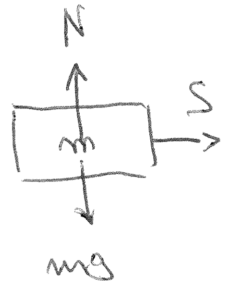
$$mg - N = m\frac{v^2}{\rho}.$$

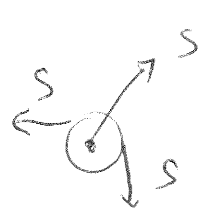
Bilen tappar kontakt med marken när $N = 0 \Rightarrow v = \sqrt{g\rho}$.

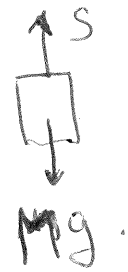
Exempel.



ingen friktion
masslös rep och trissa.







$S = ma$

$Mg - S = Ma$ (OBS! $\neq 0$.)

VARNING FÖR JÄMVIKTSTÄNKANDE!

$$\begin{cases} S = ma \\ Mg - S = Ma \end{cases}$$

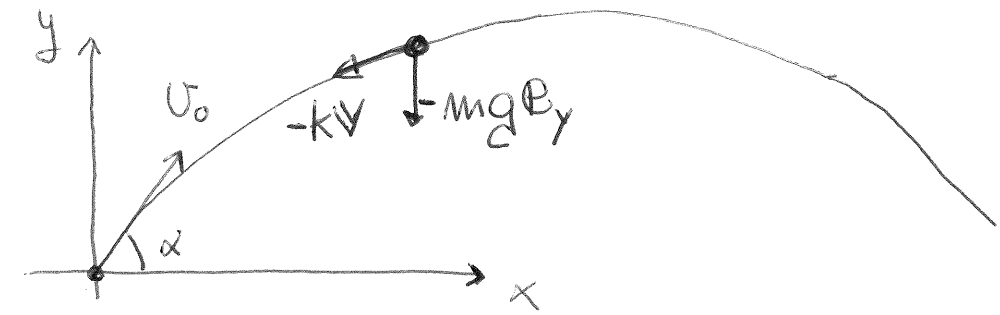
$$\Downarrow$$

$$a = \frac{M}{m+M} g$$

(Rimligt? låt $m \ll M$ eller $m \gg M$)

$$S = \frac{mM}{m+M} g$$

KASTRÖRELSE (PROJECTILE MOTION)



$x(t_0=0) = y(t_0=0) = 0$

UTAN LUFTMOTSTÅND:
$(k=0)$

$$\begin{cases} m\ddot{x} = 0 \\ m\ddot{y} = -mg \end{cases}$$

$$\Rightarrow \begin{cases} x = v_{0x} t = v_0 \cos\alpha \cdot t \\ y = v_{0y} t - \frac{1}{2} g t^2 = v_0 \sin\alpha \, t - \frac{1}{2} g t^2 \end{cases}$$

$$\Rightarrow y = -\frac{g}{2 v_0^2 \cos^2_\alpha} x^2 + \tan\alpha \, x \quad \text{(parabel)}$$

MED LUFTMOTSTÅND:
(AIR RESISTENCE)

$$\begin{cases} m\ddot{x} = -k\dot{x} & [1] \\ m\ddot{y} = -k\dot{y} - mg. & [2] \end{cases}$$

$[k] = kg/s$

$$[1]: \quad \dot{v}_x = -\frac{k}{m} v_x \Rightarrow \frac{dv_x}{v_x} = -\frac{k}{m} dt$$

$$\Rightarrow \int_{v_{x0}}^{v_x} \frac{dv_x'}{v_x'} = -\frac{k}{m} t \Rightarrow \log \frac{v_x}{v_{x0}} = -\frac{kt}{m}$$

$$\Rightarrow v_x = v_{x0} e^{-kt/m} = v_0 \cos\alpha \, e^{-kt/m}$$

$$\Rightarrow x(t) = \frac{m v_0 \cos\alpha}{k} \left(1 - e^{-kt/m}\right)$$

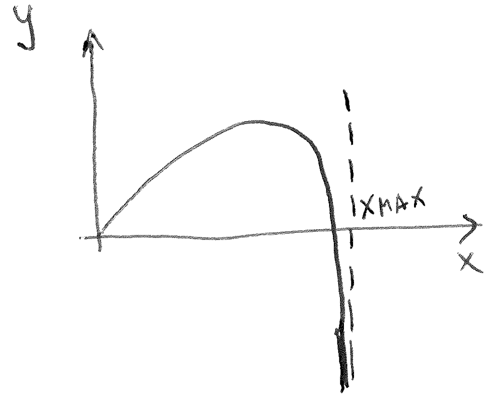
[2] $$\dot{v}_y = -\frac{k}{m} v_y - g \Rightarrow \frac{dv_y}{v_y + \frac{mg}{k}} = -\frac{k}{m} dt$$

$$\Rightarrow \log \frac{v_y + \frac{mg}{k}}{v_{y_0} + \frac{mg}{k}} = -\frac{k}{m} t$$

$$\Rightarrow v_y + \frac{mg}{k} = \left(v_{y_0} + \frac{mg}{k}\right) e^{-\frac{kt}{m}}$$

$$\Rightarrow v_y = -\frac{mg}{k} + \left(v_0 \sin\alpha + \frac{mg}{k}\right) e^{-\frac{kt}{m}}.$$

$$\Rightarrow y = -\frac{mg}{k} t + \left(v_0 \sin\alpha + \frac{mg}{k}\right) \frac{m}{k} \left(1 - e^{-\frac{kt}{m}}\right)$$



$$\lim_{t \to \infty} v_x = 0$$

$$\text{"} \quad x = \frac{m v_0 \cos\alpha}{k}$$

$$\text{"} \quad v_y = -mg/k$$

$$\text{"} \quad y = -\infty$$

GRÄNSHASTIGHETEN : $v_g = mg/k \Rightarrow k = \frac{mg}{v_g}$
(LIMITING VELOCITY)

$$x_{max} = \frac{m v_0 \cos\alpha}{k} = \frac{v_0 v_g \cos\alpha}{g}$$