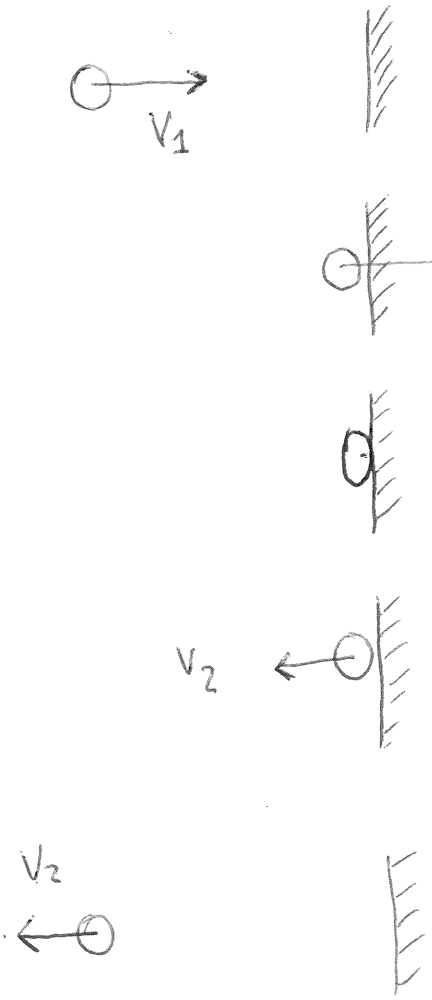
RÖRELSEMÄNGD (MOMENTUM) och IMPULS (IMPULSE)

Def: $\mathbb{P} = m\mathbb{V}$ (rörelsemängd).

Newtons II $\Rightarrow$ $\mathbb{F} = \dot{\mathbb{P}}$ (OM m är konstant!) (ej raket!).

$$\Rightarrow \underbrace{\int_{t_1}^{t_2} \mathbb{F}\, dt}_{\text{IMPULS}} = \mathbb{P}_2 - \mathbb{P}_1$$

$\mathbb{P}$ BEVARAT OM $\mathbb{F} = 0$.

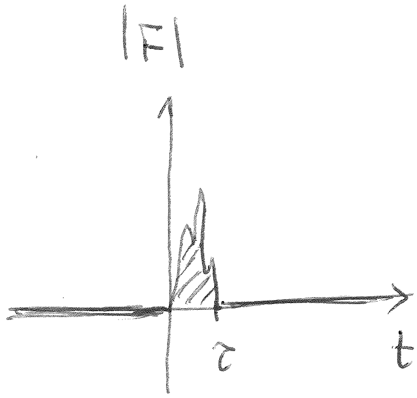


$t \ll 0$

$t = 0$

$t = \tau \simeq 0$

$t \gg 0$



RÖRELSEMÄNGD MOMENT och IMPULSMOMENT

(ANGULAR MOMENTUM) (ANGULAR IMPULSE)

Def: $\mathbf{L}_0 = \mathbf{r} \times \mathbf{P}$

VRIDMOMENT

$$\dot{\mathbf{L}}_0 = \underbrace{\cancel{\dot{\mathbf{r}} \times \mathbf{P}}}_{0} + \mathbf{r} \times \dot{\mathbf{P}} = \mathbf{r} \times \mathbf{F} \equiv \mathbf{M}_0$$

$$\Rightarrow \int_{t_1}^{t_2} \mathbf{M}_0 \, dt = \mathbf{L}_{0_2} - \mathbf{L}_{0_1}$$

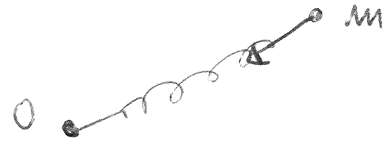
($\mathbf{L}_0$ BEVARAD OM $\mathbf{M}_0 = 0$)

OBS: I ett CENTRALKRAFTFÄLT ($\mathbf{F} \propto \mathbf{e}_r$)

är $\mathbf{M}_0 \propto \mathbf{r} \times \mathbf{e}_r = r \, \mathbf{e}_r \times \mathbf{e}_r = 0$.

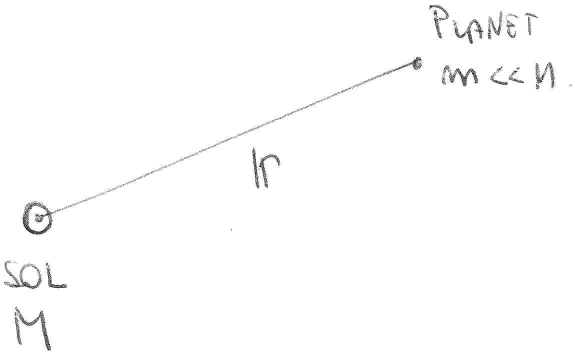
$\Rightarrow$ $\mathbf{L}_0$ bevarad.

$$\mathbf{F} = -G \frac{Mm}{r^2} \mathbf{e}_r$$



$$\mathbf{F} = -k(r - r_0) \mathbf{e}_r$$

KEPLERS PROBLEM:



$$\begin{cases} -G\dfrac{\cancel{m}M}{r^2} = \cancel{m}(\ddot{r} - r\dot{\theta}^2) \\ 0 = \cancel{m}(r\ddot{\theta} + 2\dot{r}\dot{\theta}) \end{cases}$$

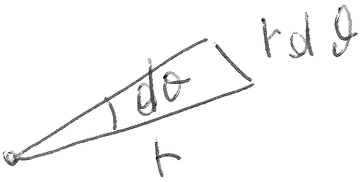
m FÖRSVINNER!

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 \Rightarrow \frac{d}{dt}(r^2\dot{\theta}) = 0$$

$\Rightarrow r^2\dot{\theta}$ KONSTANT.

OBS: $L_0 = m r v_\theta = m r^2 \dot{\theta}$

KEPLERS II "Den area som vektorn från solen till planeten översveper per sekund är konstant under rörelsen."

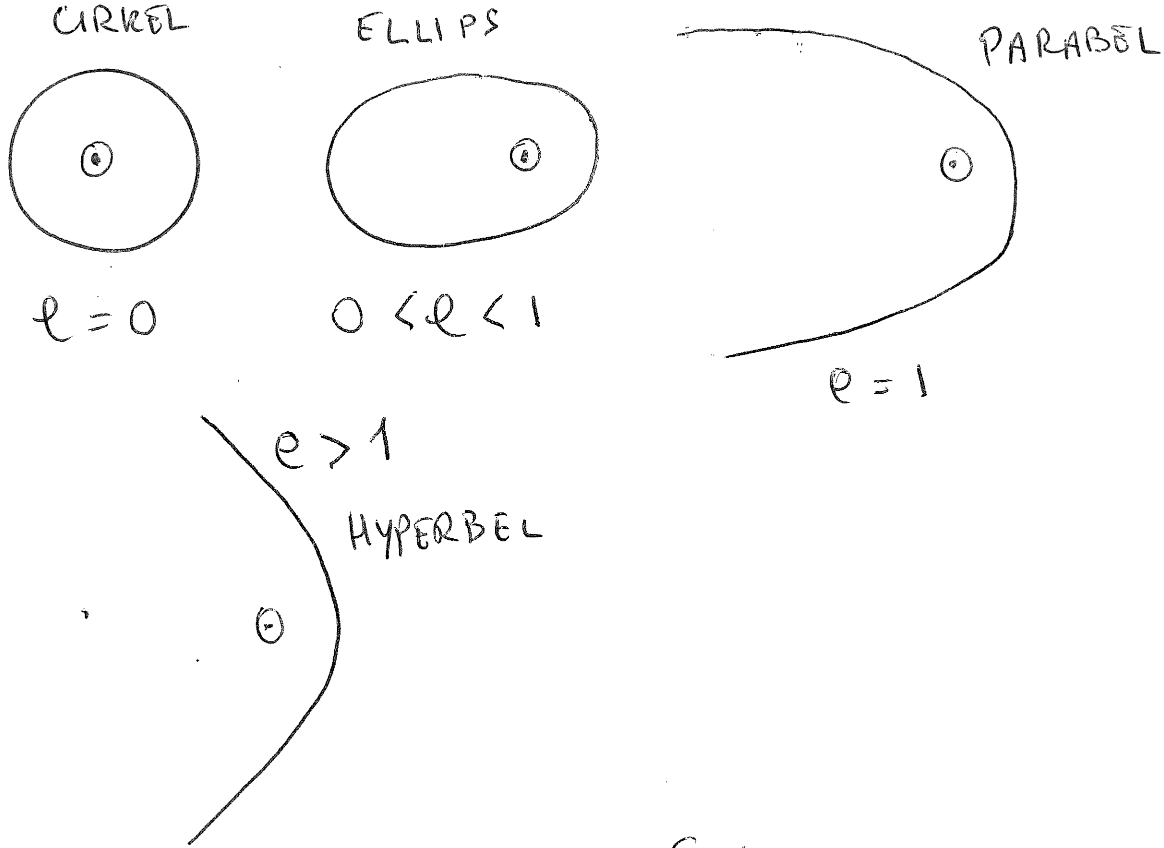


$$dA = \frac{1}{2} r \cdot r d\theta$$

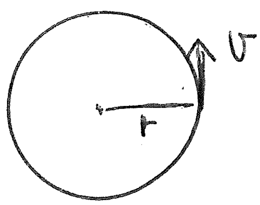
$$\Rightarrow \dot{A} = \frac{1}{2} r^2 \dot{\theta}$$

Resten av ekvationssystem kan också lösas:

$$r = \underbrace{\left(\frac{L^2}{Gm^2M}\right)}_{\text{konstant.}} \cdot \frac{1}{1+\underbrace{e}_{\text{eccentricitet.}}\cos\theta}$$



($e=0$ fall:



$$\begin{cases} L = mvr \\ m\dfrac{v^2}{r} = G\dfrac{mM}{r^2} \end{cases}$$

$$\Rightarrow r = \frac{L^2}{Gm^2M}.$$)

OBS: $E = \frac{1}{2}mv^2 - G\frac{mM}{r}$ är också bevarad $\forall e$.