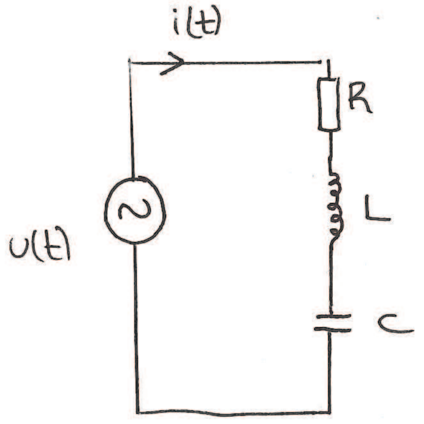
Mån LV2

Seriekrets



$U(t) = \hat{U} \sin(\omega t + \alpha)$

Effektivvärden U, I

$\Rightarrow \underbrace{P}_{\text{medeleffekt}} = UI \cos\varphi$

$U_R = RI$

$U_L = X_L I = \omega L I$ — spänning $\frac{\pi}{2}$ (90)° före strömmen

$U_C = X_C I = \frac{1}{\omega C} I$ — spänning $\frac{\pi}{2}$ (90°) efter strömmen

För att bestämma U_i addera vektoriellt!

jω-~~omega~~-metoden

Komplexa tal $i \equiv j$

Ett tidsoberoende problem

1) Kirchhoffs laga gäller ALLTID

2) stationär sinusformad spänning

3) Linjära kretsar ($\equiv$ Ohms lag gäller)

$\hookrightarrow u(t) = \hat{U} \sin(\omega t + \alpha) \rightarrow i(t) = \hat{I} \sin(\omega t + \beta)$ (1)

Komplexa tal

$$\bar{z} = a + bj = |\bar{z}|e^{j\alpha} = |\bar{z}|(\cos\alpha + j\sin\alpha)$$

$$j = \sqrt{-1}$$

$$|\bar{z}| = \sqrt{a^2+b^2}, \quad e^{j(\alpha+\beta)} = e^{j\alpha}e^{j\beta}, \quad e^{j(\omega t+\alpha)} = \cos(\omega t+\alpha) + j\sin(\omega t+\alpha)$$

Komplex spänning $\bar{U}$ resp komplex ström $\bar{I}$

$$\bar{U} = \hat{U}\Big(\cos(\omega t+\alpha) + j\sin(\omega t+\alpha)\Big) = \hat{U}e^{j\alpha}e^{j\omega t} \qquad (2)$$

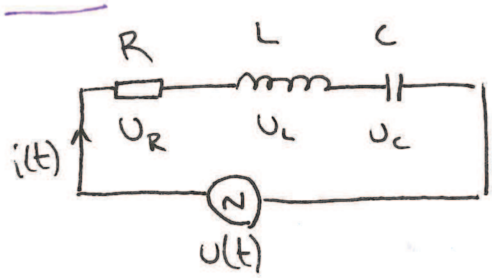
$$\bar{I} = \hat{I}\Big(\cos(\omega t+\beta) + j\sin(\omega t+\beta)\Big) = \hat{I}e^{j\beta}e^{j\omega t} \qquad (3)$$

Jämför (1) och (2) som ger

$$u(t) = Re(\bar{U}) = \hat{U}\cos(\omega t+\alpha)$$

$$i(t) = Re(\bar{I})$$

Ex.



Enligt Kirchhoffs lagar.

$$u(t) = U_R + U_L + U_C$$

$$u(t) = R\cdot i(t) + L\frac{di}{dt} + \frac{1}{C}\int^{t} i(t)\,dt$$

Derivering & integrering av (1) ger

$$\hat{U}e^{j\alpha}\cancel{e^{j\omega t}} = R\hat{I}e^{j\beta}\cancel{e^{j\omega t}} + \hat{I}j\omega Le^{j\beta}\cancel{e^{j\omega t}} + \frac{1}{j\omega C}e^{j\beta}\cancel{e^{j\omega t}}\hat{I}$$

$$\hat{U}e^{j\alpha} = R\hat{I}e^{j\beta} + j\omega Le^{j\beta}\hat{I} + \frac{\hat{I}}{j\omega C}e^{j\beta}$$

$$\hat{U}e^{j\beta} = \hat{I}e^{j\beta}\underbrace{\left(R + j\left(\omega L - \frac{1}{\omega C}\right)\right)}_{\bar{Z}\ \text{Impedans}} \qquad \left\{\frac{1}{j} = -j\right\}$$

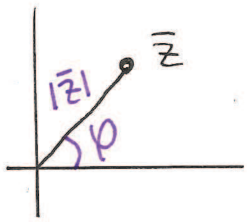
Def. $\bar{Z}$ (Impedans):

$$\bar{Z} = \frac{\bar{U}}{\bar{I}}$$

$$\left\{\begin{array}{l}\bar{U} = \hat{U}e^{j\alpha} \\ \bar{I} = \hat{I}e^{j\beta}\end{array}\right\}$$ Transformer för $u(t)$ resp. $i(t)$

$$\hat{U}e^{j\alpha} = \hat{I}e^{j\beta}\bar{Z}$$

$$\bar{Z} = \frac{\hat{U}e^{j\alpha}}{\hat{I}e^{j\beta}} = |\bar{Z}|e^{j\varphi} \quad ; \quad |\bar{Z}| = \frac{\hat{U}}{\hat{I}}, \quad e^{j\varphi} = e^{j(\alpha-\beta)}$$



Lösningsgång

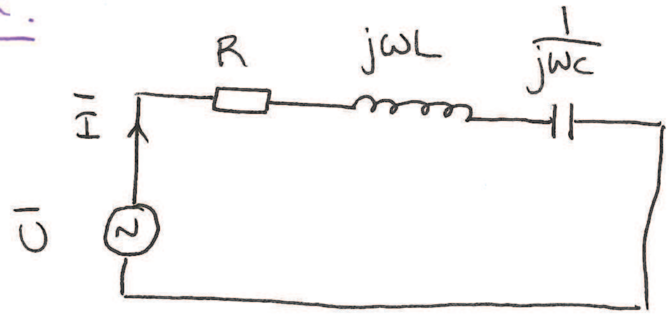
1. Inför transformer för ström och spänning

$$\bar{U} = \hat{U}e^{j\alpha} \qquad u(t) = \hat{U}\cos(\omega t + \alpha)$$

$$\bar{I} = \hat{I}e^{j\beta} \qquad i(t) = \hat{I}\cos(\omega t + \beta)$$

2. $$\boxed{\begin{array}{l} R \rightarrow R \\ L \rightarrow j\omega L \\ C \rightarrow \dfrac{1}{j\omega C}\end{array}}$$

Ex.



$$\bar{Z} = R + jX$$

$$\bar{Z} = \frac{\bar{U}}{\bar{I}} \qquad \underbrace{\bar{U} = \bar{Z}\bar{I}}_{\text{ej ohms lag}}$$

$$\bar{U} = \bar{I}\left(R + j\omega L + \frac{1}{j\omega C}\right)$$

$$\cancel{\bar{I} = \frac{\bar{U}}{R + j\omega\left(L - \frac{1}{C}\right)}} \qquad \bar{I} = \frac{\bar{U}}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\bar{I} = \hat{I}e^{j\beta} = \frac{\hat{U}}{|\bar{Z}|e^{j\varphi}} = \frac{\hat{U}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}\, e^{-j\varphi}$$

$$\hat{I} = I\sqrt{2} = \frac{\hat{U}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\arctan \frac{\text{Im}}{\text{Re}} = \varphi \qquad \arctan \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}$$

$$|\bar{Z}| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

Resonans: $\omega L - \frac{1}{\omega C} = 0 \iff \omega L = \frac{1}{\omega C}$

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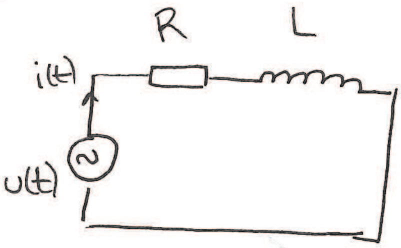
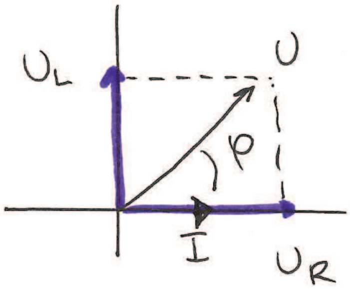
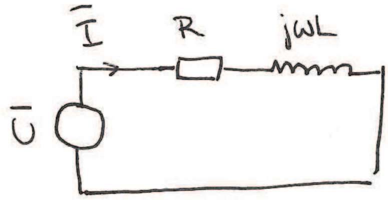




$$u(t) = \hat{U}\cos\omega t \equiv \hat{U}\sin\omega t$$???

$$\bar{U} = \hat{U}$$
$$\bar{I} = \hat{I}e^{j\varphi}$$



$$\bar{Z} = \bar{Z}_R + \bar{Z}_L = R + j\omega L$$

$$\bar{I} = \frac{\bar{U}}{\bar{Z}} = \frac{\bar{U}}{R + j\omega L} = \hat{I}e^{j\varphi}$$

$$\hat{I} = |\bar{I}| = \frac{\hat{U}}{\sqrt{R^2 + (\omega L)^2}}$$

$$\bar{U} = \hat{U}$$

$$\varphi = \arctan\frac{Im}{Re} = \arctan\frac{\omega L}{R}$$

Impedans (Ω)

Serie: $Z_1 + Z_2 + Z_3 + \ldots = Z$

parallell: $\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \ldots$

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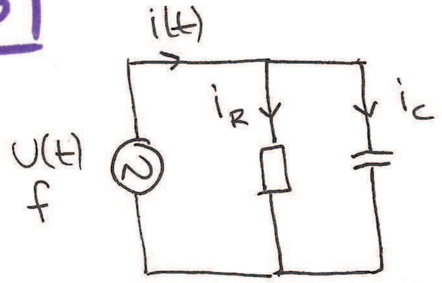

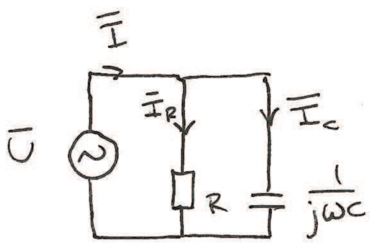

Bestam t och φ för kretsen

Totalström: $\bar{I} = \bar{I}_R + \bar{I}_c = \dfrac{\bar{U}}{R} + \dfrac{\bar{U}}{(j\omega C)^{-1}} = \dfrac{\bar{U}}{R} + j\omega C\bar{U}$

$$\bar{I} = \bar{U}\left(\frac{1}{R} + j\omega C\right)$$

$$\bar{U} = \bar{Z}\bar{I} \iff \bar{Z} = \frac{\bar{U}}{\bar{I}}$$

$$\bar{Z} = \frac{\bar{U}}{\bar{I}} = \frac{1}{\frac{1}{R} + j\omega C} = \frac{R}{1 + j\omega RC}$$

$$\varphi = \arg \bar{Z} = \arg R - \arg(1 + j\omega RC) = 0 - \arctan(\omega RC)$$

$$= -\arctan(\omega RC)$$