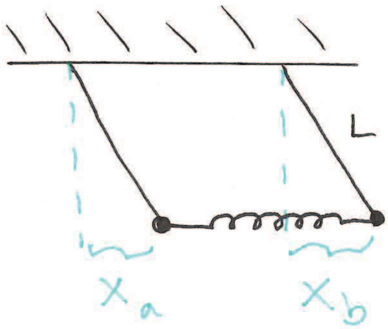
Kopplade oscillatorer

Tors Lv4

(1)



$(x_a = x_b)$

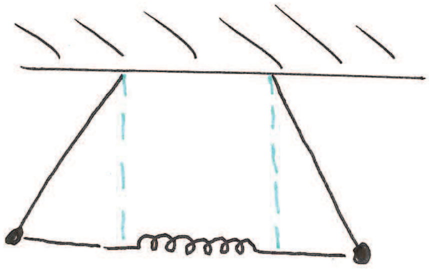
$$m\ddot{x}_a + \omega_1^2 x_a = 0$$

$$\omega_1 = \sqrt{g/L}$$

$$\varphi = 0$$

$$x_a = A\cos\omega_1 t = x_b$$

(2)



$$x_a = -x_b$$

$$m\ddot{x}_a = -\frac{mg}{L}x_a - 2kx_a$$

$$\ddot{x}_a = \omega_2^2 x_a = 0$$

$$\omega_2 = \sqrt{\frac{g}{L} + \frac{2k}{m}}$$

$$x_a = B\cos\omega_2 t$$

$$x_b = -B\cos\omega_2 t$$

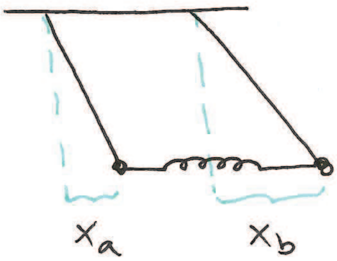
$$= B\cos(\omega_2 t + \pi)$$

$(\omega_2 > \omega_1)$

Normala moder och normala frekv.

- Båda m svänger med samma ω
- Varje m utför SHM
- $\varphi = 0$ eller π
- Samma mod behålls hela tiden

③ $x_a \neq x_b$



$$\begin{cases} \ddot{x}_a + \frac{g}{L}x_a + \frac{k}{m}(x_a - x_b) = 0 \\ \ddot{x}_b + \frac{g}{L}x_b - \frac{k}{m}(x_a - x_b) = 0 \end{cases}$$

Addition $$(\ddot{x}_a + \ddot{x}_b) + \frac{g}{L}(x_a + x_b) = 0$$

$$\omega = \sqrt{g/L} = \omega_1$$

Subtraktion $$(\ddot{x}_a - \ddot{x}_b) + \left(\frac{g}{L} + \frac{2k}{m}\right)(x_a + x_b) = 0$$

$$\omega = \sqrt{\left(\frac{g}{L}\right) + \left(\frac{2k}{m}\right)} = \omega_2$$

Substitution $$q_1 = x_a + x_b$$

$$q_2 = x_a - x_b$$

$$\ddot{q}_1 + \omega_1^2 q_1 = 0$$

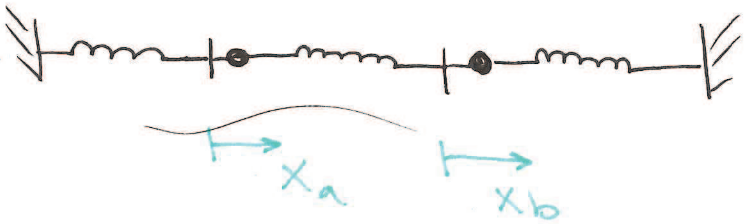
$$\ddot{q}_2 + \omega_2^2 q_2 = 0$$

q_1 i q_2 normala koordinater

Oberoende oscillationer $$\begin{cases} q_1 = c_1 \cos(\omega_1 t + \varphi_1) \\ q_2 = c_2 \cos(\omega_2 t + \varphi_2) \end{cases}$$

$$x_a = \frac{1}{2}(q_1 + q_2) = \frac{1}{2}\left(c_1\cos(\omega_1 t + \varphi_1) + c_2\cos(\omega_2 t + \varphi_2)\right)$$

$$x_b = \frac{1}{2}(q_1 - q_2) = \frac{1}{2}\left(c_1\cos(\omega_1 t + \varphi_1) - c_2\cos(\omega_2 t + \varphi_2)\right)$$



$$m\ddot{x}_a = -kx_a + kx_b - kx_a = kx_b - 2kx_a$$

$$m\ddot{x}_b = -kx_b + kx_a - kx_b = kx_a - 2kx_b$$

$$x_a = A\cos\omega t$$

$$x_b = B\cos\omega t$$

$$-Am\omega^2\cos\omega t - kB\cos\omega t - 2kA\cos\omega t \quad (1)$$

$$-Bm\omega^2\cos\omega t = kA\cos\omega t - 2kB\cos\omega t \quad (2)$$

$$\left.\begin{array}{l} (1) \rightarrow \dfrac{A}{B} = \dfrac{k}{2k - m\omega^2} \quad (3) \\[2ex] (2) \rightarrow \dfrac{A}{B} = \dfrac{2k - m\omega^2}{k} \quad (4) \end{array}\right\} = (2k - m\omega^2)^2 = k^2$$

$\omega^2 = k/m$ in i (3) ger $A = B \rightarrow x_a = A\cos\omega_1 t$

$\omega^2 = 3k/m$ in i (4) ger $A = -B \rightarrow x_b = -A\cos\omega_2 t$

Allmänt fall: Spp: $x_a = c_1\cos\omega_1 t + c_2\cos\omega_2 t$

$$x_b = c_1\cos\omega_1 t - c_2\cos\omega_2 t$$