Mån LV6

Total energi hos vibrerande sträng

Energi för normalmod

$$E_n = \frac{1}{4}\mu L A_n^2 \omega_n^2 \underbrace{\left(\sin^2 \omega_n t + \cos^2 \omega_n t\right)}_{=1}$$

→ Total energi för en stående våg

$$E = \sum_n E_n = \frac{1}{4}\mu L \sum_n A_n^2 \omega_n^2$$

Att normalmodernas energi är additiva betyder att de är oberoende ("okopplade")

5.8

$L = 25 \text{ m}$

$m = 0.1 \text{ kg}$

$T = 10 \text{ N}$

$f = 25 \text{ Hz}$

$A = 15 \cdot 10^{-3} \text{ m}$

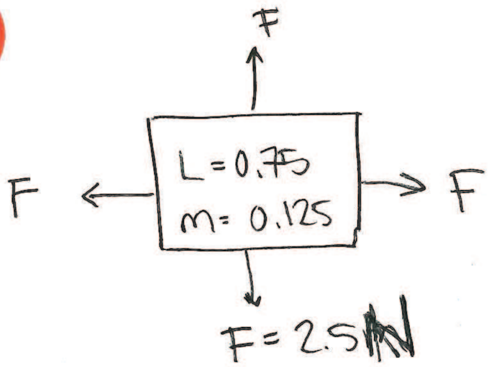
i) $V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{m/L}} = \sqrt{\frac{TL}{m}} = \sqrt{\frac{25 \cdot 10}{0.1}} = 50 \text{ m/s}$

ii) $\dot{y}_{max} = \left(\frac{\partial y}{\partial t}\right)_{max}$?

$y(x,t) = A\cos(kx - \omega t) \rightarrow \dot{y} = \omega A \sin(kx - \omega t)$

$\dot{y}_{max} = \omega A = 2\pi f A = \ldots = 2.4 \ m/s$

b)



$$V = \sqrt{\frac{S}{\sigma}} = \sqrt{\frac{F/L}{m/L^2}} = \sqrt{\frac{FL}{m}} = \ldots = 3.9 \ m/s$$

5.13

$\mu_1 = 1 \cdot 10^{-8} \ kg/m$

$\mu_2 = 4 \cdot 10^{-8} \ kg/m$

$A_1 = 0.030 \ m$

$\lambda_1 = 0.25 \ m$

a) $$\lambda = \frac{V}{f} = \frac{\sqrt{T/\mu}}{f} = \frac{1}{f}\sqrt{\frac{T}{\mu}}$$

$$\left.\begin{aligned} \lambda_1 &= \frac{1}{f}\sqrt{T/\mu_1} \\ \lambda_2 &= \frac{1}{f}\sqrt{T/\mu_2} \end{aligned}\right\} \Longrightarrow \frac{\lambda_2}{\lambda_1} = \sqrt{\frac{\mu_1}{\mu_2}}$$

$$\lambda_2 = \lambda_1 \sqrt{\frac{\mu_1}{\mu_2}} = \ldots = \frac{0.25}{2} \ m \ (.125 \ m)$$

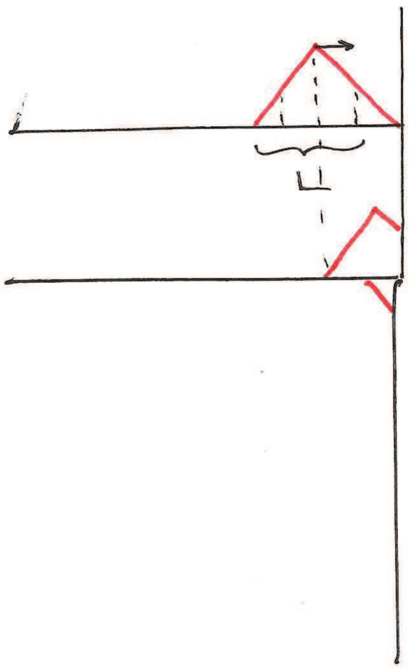
$$T_{12} = \frac{A_2}{A_1} = \frac{2k_1}{k_1+k_2} = \frac{2 \cdot \frac{2\pi}{\lambda_1}}{\frac{2\pi}{\lambda_1} + \frac{2\pi}{\lambda_2}} = \dots = \frac{2}{3}$$

$$\rightarrow A_2 = \frac{2}{3} A_1 = 0.020 \text{ m}$$

b) $$R_{12} = \frac{B_1}{A_1} = \frac{k_1 - k_2}{k_1 + k_2} = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} = \dots = -\frac{1}{3}$$

$$I \sim A_{12} = \frac{I_R}{I_I} = \left(\frac{A_R}{A_I}\right)^2 = R_{12}^2 = \frac{1}{9}$$

En triangelformad puls (längd L) breder ut sig längs en sträng och reflekteras mot en fixerad ~~sträng~~ ände. utseende då $\frac{L}{4}$, $\frac{2L}{4}$, $\frac{3L}{4}$ och L av pulsen har reflekterats?



6.3 $\lambda_n = 0.55\,m$

$\lambda_{n+1} = 0.44\ m$

a) $L = \frac{n\lambda_n}{2} = \frac{(n+1)\lambda_{n+1}}{2} \Rightarrow n\lambda_n = (n+1)\lambda_{n+1}$

$n(\lambda_n - \lambda_{n+1}) = \lambda_{n+1}$

$n = \frac{\lambda_{n+1}}{\lambda_n - \lambda_{n+1}} = \frac{0.44}{0.55 - 0.44} = 4$

$L = \frac{4 \cdot 0.55}{2} = 1.1\,m$

(b) $c = 3 \cdot 10^8\ m/s$

$\lambda/2 = 0.005\ m$

$f = \frac{c}{\lambda} = \cancel{\frac{3 \cdot 10^8}{\ }} = \frac{3 \cdot 10^8}{2 \cdot 0.005} = 3 \cdot 10^{10}\ Hz$

6.12 $f(x) = \alpha x$ i intervallet $x \in [0, L]$

$f(x) = \sum_n A_n \sin\left(\frac{n\pi}{L} x\right)$

$A_n = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{n\pi}{L} x\right) dx \qquad n = 1, 2, 3, \ldots$

$$A_n = \frac{2}{L}\int_0^L \alpha x \sin\left(\frac{\pi n}{L}x\right)dx = \frac{2\alpha}{L}\int_0^L x\sin\left(\frac{n\pi}{L}x\right)dx$$

Trig. integral: $\int x \sin ax = \frac{1}{a^2}\sin ax - \frac{x}{a}\cos ax$

$$A_n = \frac{2\alpha}{L}\left[\left(\frac{L}{n\pi}\right)^2 \sin\left(\frac{n\pi}{L}x\right) - x\left(\frac{L}{n\pi}\right)\cos\left(\frac{n\pi}{L}x\right)\right]_0^L$$

$$= \frac{2\alpha}{L}\left(0 - \frac{L^2}{n\pi}\cos(n\pi) - 0 + 0\right)$$

$$= -\frac{2\alpha L}{n\pi}\cos(n\pi) = \cancel{\frac{2\alpha L}{n\pi}} = \frac{2\alpha L}{\pi}\,\frac{(-1)^{n+1}}{n}$$

$$\left.\begin{aligned} A_1 &= \frac{2\alpha L}{\pi} \\ A_2 &= -\frac{1}{2}A_1 \\ A_3 &= -\frac{1}{4}A_1 \end{aligned}\right\} \quad f(x) = \sum_n A_n \sin\left(\frac{n\pi}{L}x\right) =$$

$$= \frac{2\alpha L}{\pi}\left[\sin\left(\frac{\pi}{L}x\right) - \frac{1}{2}\sin\left(\frac{2\pi}{L}x\right) + \frac{1}{3}\sin\left(\frac{3\pi}{L}x\right) \ldots\ldots\right]$$