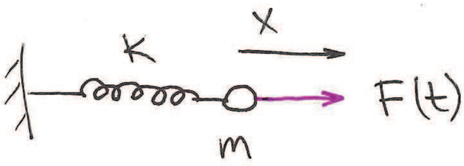
Sammanfattning 2

Tors LV 7

Påtvingade svängningar

1) odämpade



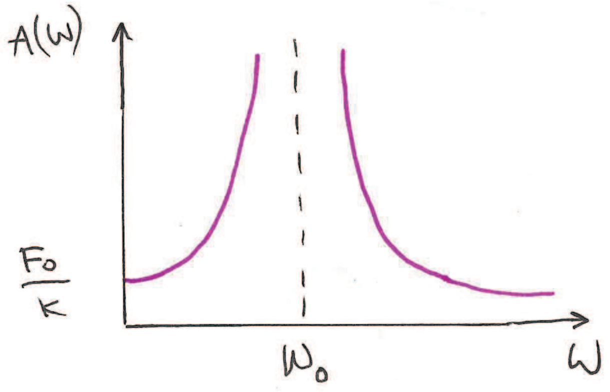
$$m\ddot{x} + kx = F_0 \cos \omega t$$

$$x(t) = A(\omega) \cos(\omega t - \delta)$$

$A(\omega)$ = fysisk amplitud, beroende av frekvens

~~δ drivande kraften~~

δ fas-skillnad mellan drivande kraft och förflyttningen x.



$$A(\omega) = \frac{a}{\left(1 - \frac{\omega^2}{\omega_0^2}\right)}$$

2) dämpade

dämpande kraft ~ proportionell mot hastigheten av massan.

$$m\ddot{x} + b\dot{x} + kx = F_0 \cos \omega t$$

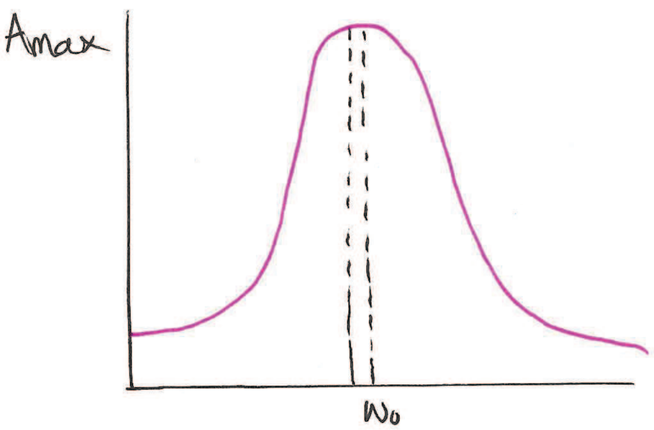
$$x(t) = A(\omega) \cos(\omega t - \delta) \qquad x(t) = x_h + x_p$$

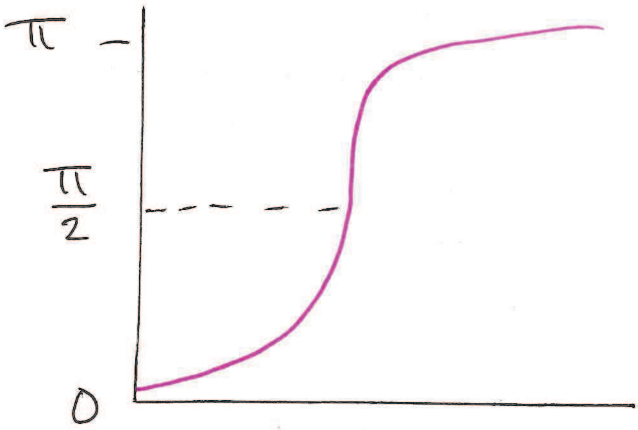
homogen partikulär

$$A(\omega) = \frac{F_0/m}{\left((\omega_0^2 - \omega^2)^2 + \omega^2 \gamma^2\right)^{\frac{1}{2}}}, \quad \gamma = \frac{b}{m}$$

$$\tan \delta = \frac{\omega \gamma}{(\omega_0^2 - \omega^2)}$$

lite fel...

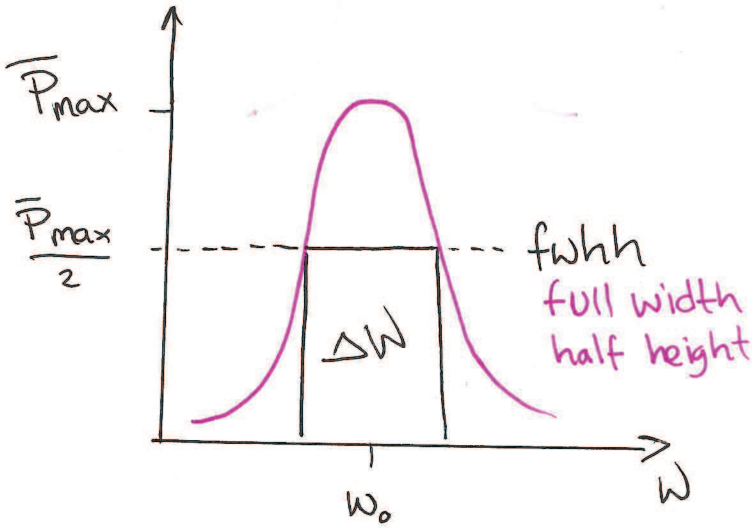


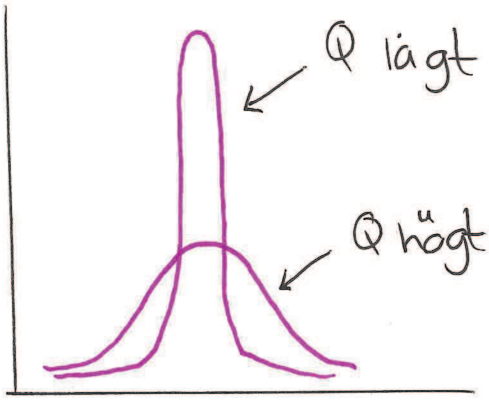


Effekt

$$p(t) = b\,v(t) \cdot v(t)$$

$$\overline{P}(\omega) = \frac{1}{T} \int_0^T p(t)\,dt$$

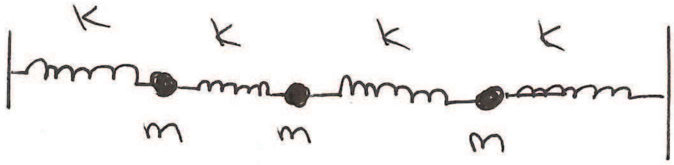




$$Q = \frac{\omega_0}{\omega_{fwhm}} = \frac{\omega_0}{\Delta\omega}$$

Kopplade svängningar

Flera kroppar – oscillatorer – kopplade tillsammans



Ett system av kopplade diff. ekvationer

Variabelbyte: normala koordinater ⟶ normala moder

Dispersion

uppstår när vågor fortskrider i ett dispersivt medium tex, vatten.

(icke-dispersivt medium – luft)

Hastigheten av en våg beror på ~~den~~ dess frekvens

$$\begin{cases} \omega = \omega(k) \\ k = \frac{2\pi}{\lambda} \\ v = \lambda \cdot f \end{cases}$$

fashastighet, grupphastighet:

dispersivt $v \neq v_g \rightarrow v_g = \frac{d\omega}{dk}$

icke-disp. $v = v_g \Rightarrow v = \frac{\omega}{k} = \text{konst.}$

Ex. e EM-strålning

$$v = \frac{\omega}{k} = \frac{c}{\sqrt{1 - \frac{\omega_0^2}{\omega^2}}}$$

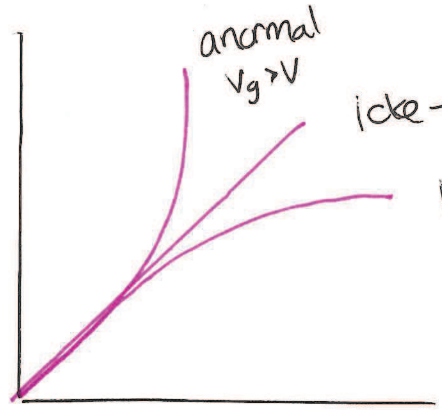
$v \geq c$

$v \cdot v_g = c^2$

$$v_g = \frac{d\omega}{dk} = c\sqrt{1 - \frac{\omega_0^2}{\omega^2}}$$

$v_g \leq c$

signaler fortskrider <u>alltid</u> med v_g



$$v_g = \frac{d\omega}{dk}$$

$$v = \frac{\omega}{k} = \text{konst}$$

$$v = \lambda f$$

8.6 stående våg + dispersion

$$\omega^2 = gk + \frac{Sk^3}{\rho}$$

Avstånd mellan bukar : 0.25 mm

$f = 1.35 \cdot 10^3$ Hz

$\lambda = 2 \cdot 0.25 = 0.5$ mm $(0.5 \cdot 10^{-3}$ m$)$

$$V = \lambda f = 0.5 \cdot 1.35 = 0.675 \frac{m}{s}$$

(a) Sökt: ytspänning, S

$$\omega^2 = \underbrace{gk}_{\text{försumma}} + \frac{Sk^3}{\rho}$$

ty ytsp. >> gravitation $\quad \omega^2 = \frac{Sk^3}{\rho}$

$$\left.\begin{array}{l} V = \frac{\omega}{k} \\ \omega^2 = \frac{Sk^3}{\rho} \end{array}\right\} \Rightarrow V = \sqrt{\frac{Sk^3}{\rho}}$$

$$S = \frac{V^2 \rho}{k} = \frac{V^2 \rho \lambda}{2\pi} = 0.49 \frac{N}{m}$$

(b) V_g?

$$V_g = \frac{d\omega}{dk} = \frac{d}{dk}\left(\sqrt{\frac{S}{\rho}}\, k^{\frac{3}{2}}\right) = \frac{3}{2}\sqrt{\frac{S}{\rho}}\, k^{\frac{3}{2}-1} = \frac{3}{2}\sqrt{\frac{S}{\rho}}\sqrt{k}$$

$$= \frac{3}{2} V \simeq 1.0 \ m/s$$

$$v = \frac{w}{k}$$

$$w^2 = \frac{Sk^3}{\rho} = (vk)^2 = v^2 k^2$$

$$v^2 = \frac{Sk}{\rho} \qquad v = \sqrt{\frac{Sk}{\rho}} \qquad S = \frac{v^2 \rho}{k}$$