Demoövn. 3

Fre LV3

EÖ6 a) Bestäm F-transf. av $\widehat{\dfrac{t}{(t^2+a^2)^2}}$

Vet att $\underbrace{\widehat{\dfrac{1}{t^2+a^2}}}_{(*)} = \dfrac{\pi}{a}\exp(-|a|\xi)$ { Obs! Fel! Abs. belopp på ξ, inte a! ^^

Deriverar vi $(*)$ så får vi:

$$-\frac{1}{2}\,\widehat{\frac{d}{dt}\frac{1}{t^2+a^2}} = -\frac{1}{2}i\xi\,\widehat{\frac{1}{t^2+a^2}}(\xi) =$$

$$= -\frac{i\xi}{2}\frac{\pi}{a}\exp(-a|\xi|)$$ F-transf. som söktes!

b) Bestäm F-transf. av $\widehat{\dfrac{1}{(t^2+a^2)^2}}$

$$\widehat{\frac{1}{t^2+a^2}\frac{1}{t^2+a^2}} = \frac{1}{2\pi}\,\widehat{\frac{1}{t^2+a^2}} * \widehat{\frac{1}{t^2+a^2}} =$$

$$= \frac{1}{2\pi}\cdot\frac{\pi^2}{a^2}\exp(-a|\xi|) * \exp(-a|\xi|) =$$

$$= \frac{\pi}{2a^2}\int_{-\infty}^{+\infty}\exp(-a|\xi-\eta|-a|\eta|)\,d\eta =$$

$$= \frac{\pi}{2a^2}\left(\int_{-\infty}^{0}\ldots + \int_{0}^{\xi}\ldots + \int_{\xi}^{+\infty}\ldots\right)$$ om $\xi \geq 0$ jämn

Alt. (bättre)

$$\widehat{\frac{1}{(t^2+a^2)^2}} \quad \text{Derivera map } \xi\ !$$

$$i\frac{d}{d\xi}\widehat{\frac{1}{(t^2+a^2)^2}}(\xi) = \widehat{\frac{t}{(t^2+a^2)^2}}(\xi) = -\frac{\pi i}{2a}\xi\exp(-a|\xi|)$$

$$\widehat{\frac{1}{(t^2+a^2)^2}}(\xi) = +\frac{\pi \cancel{i}}{2a}\underbrace{\int_{\xi}^{+\infty}\xi'\exp(-a|\xi'|)\,d\xi'}_{(\#)} + C$$

$$(\#) = \left[-\frac{1}{a}\xi'\exp(-a\xi')\right]_{\xi}^{\infty} + \frac{1}{a}\int_{+\xi}^{\infty}\exp(-a\xi')\,d\xi$$

$$= \frac{\xi}{a}\exp(-a\xi) + \frac{1}{a^2}\exp(-a\xi) =$$

$$= \frac{\pi}{2}\left(\frac{\xi}{a^2} + \frac{1}{a^3}\right)\exp(-a\xi) \qquad \text{om } \xi > 0\ ! \quad \text{jämn}$$

$$\frac{\pi}{2}\left(\frac{|\xi|}{a^2} + \frac{1}{a^3}\right)\exp(-a|\xi|) \cancel{+} C$$

Alt. (bäst)

$$\int \frac{\exp(-\xi t)}{t^2+a^2} = \frac{\pi}{a}\exp(-a|\xi|)$$

Derivera map 2:

$$\frac{d}{da} = -\int 2a \exp(-\xi t) = -\frac{\pi}{a^2}\int$$

Derivera map A!

$$\frac{d}{da} \; -\int_{-\infty}^{\infty} \frac{2a\exp(i\xi t)}{(t^2+a^2)^2}\,dt = -\frac{\pi}{a^2}\exp(-a|\xi|) - \frac{\pi}{a}\exp(-a|\xi|)$$

$$\widehat{\frac{1}{t^2+a^2}} = \frac{1}{2a}\left(\frac{\pi^2}{2}\exp(-ia|\xi|) + \frac{\pi}{2}\exp(-a|\xi|)\,\xi\right.$$

Ö 7

$$\hat{f}(\omega) = \frac{\omega}{1+\omega^4}$$

a) $$\int_{-\infty}^{\infty} t f(t)\,dt = \widehat{tf(t)}(0) = i\frac{d}{d\omega}\hat{f}(\omega=0) =$$

$$i\frac{d}{d\omega}\frac{\omega}{(1+\omega^4)}\Bigg|_{\omega=0}$$

b) $$f'(0) = \frac{1}{2\pi}\int \hat{f}(\omega)\cancel{d\omega} = \frac{1}{2\pi}\int i\omega\,\hat{f}(\omega)\,d\omega =$$

$$= \frac{i}{2\pi}\int_{-\infty}^{+\infty}\frac{\omega}{1+\omega^4}\,d\omega$$

EÖ19

$N \,,\, (x_n)_0^{N-1}$

$$x_n = \begin{cases} 1 & n = 0, \dots, k-1 \\ 0 & n = k, \dots, N-1 \end{cases}$$

$$\hat{x}_m = \sum_{n=0}^{N-1} x_n \exp\left(-i \frac{2\pi}{N} mn\right) = \underbrace{\sum_{n=0}^{k-1} \exp\left(-i \frac{2\pi}{N} mn\right)}_{\text{ändlig geom. summa}} =$$

$$= \frac{1 - \exp\left(-i \frac{2\pi}{N} mk\right)}{1 - \exp\left(-i \frac{2\pi}{N} m\right)} \quad \text{för } m \neq 0$$

$$= \frac{\exp\left(-i \frac{\pi}{N} k\right)}{\exp\left(-i \frac{\pi}{N} m\right)} \frac{\exp\left(+i \frac{\pi}{N} mk\right) - \exp\left(-i \frac{\pi}{N} mk\right)}{\exp\left(+i \frac{\pi}{N} m\right) - \exp\left(-i \frac{\pi}{N} m\right)} =$$

$$= \frac{\sin\left(\frac{\pi}{N} mk\right)}{\sin\left(\frac{\pi}{N} m\right)}$$

$$\sum_{\mu=1}^{N-1} \frac{1 - \cos \frac{2\pi \mu k}{N}}{1 - \cos \frac{2\pi \mu}{N}} =$$

$$\left\lceil \text{Parseval: } \sum_{n=0}^{N-1} |x_n|^2 = \frac{1}{N} \sum_{m=0}^{N-1} |\hat{x}_m|^2 \right\rfloor$$

$$= \left\{ 1 - \cos 2x = 2\sin^2 x \right\} = \sum_{\mu=1}^{N-1} \frac{\sin^2 \frac{\pi \mu k}{N}}{\sin^2 \frac{\pi \mu}{N}} =$$

$$= \sum_{m=1}^{N-1} |\hat{X}_m|^2 = \sum_{m=0}^{N-1} |\hat{X}_m|^2 - k^2 = N \sum_{n=0}^{N-1} |X_n|^2 - k^2 =$$

$$= Nk - k^2$$