Forts. från föreläsning mån. LV3.

Mån LV3

Ex. Om insignalen $x(t) = \theta(t)e^{-2t}$

Heavisidefkn. $\theta(t) = H(t) = \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases}$

ger utsignal $y(t) = \theta(t)e^{-3t}$ så är

$$\hat{x}(\omega) = \int_0^\infty e^{-2t}e^{-i\omega t}\,dt = \frac{1}{2+i\omega}, \quad \hat{y}(\omega) = \frac{1}{3+i\omega}$$

Systemfunktionen: $\hat{h}(\omega) = \frac{\hat{y}(\omega)}{\hat{x}(\omega)} = \frac{2+i\omega+1-1}{3+i\omega} = 1 - \underbrace{\frac{1}{3+i\omega}}_{\hat{y}(\omega)} =$

$$\hat{h}(\omega) = \hat{\delta} - \widehat{\theta(t)e^{-3t}}$$

Impulssvaret: $h(t) = \delta(t) - \theta(t)e^{-3t}$ kausal?

Ja! Ty $h(t) = 0$ för $t<0$.

EÖ66 $F = f*f = \chi_{(-a,a)} * \chi_{(-a,a)}$

$F = 2a - x$ om $x \leq 2a$

$$\begin{cases} F = 2a - |x|, & |x| \leq 2a \\ F = 0, & |x| \geq 2a \end{cases}$$

Alt. $F(x) = \int \chi_{(-a,a)}(y)\chi_{(-a,a)}(x-y)\,dy =$

$$= \int \chi_{-a<y<a}\, \chi_{\underbrace{-a<x-y<a}_{x-a<y<x+a}}\, dy = \int_{\substack{-a<y<a \\ x-a<y<x+a}} 1\, dy =$$

$$= \ldots = \max(2a-|x|, 0)$$

7.2.13 (a) $\displaystyle\int_{-\infty}^{+\infty} \frac{\sin at \sin bt}{t^2}\,dt = \int \frac{\sin at}{t}\,\frac{\sin bt}{t}\,dt =$

$$\frac{\sin at}{t} = \frac{1}{2}\hat{\chi}_{(-a,a)}(t)$$

$$= \frac{1}{4}\int \chi_{(-a,a)}(\xi)\,\overline{\hat{\chi}_{(-b,b)}(\xi)}\,d\xi = \frac{1}{4}2\pi\int \chi_{(-a,a)}(x)\chi_{(-b,b)}(x)\,dx$$

$$= \frac{\pi}{2}\cdot 2\min(a,b)$$

(b) $\displaystyle\int_{-\infty}^{+\infty} \frac{t^2}{(t^2+a^2)(t^2+b^2)}\,dt = \int \frac{t}{(t^2+a^2)}\,\frac{t}{(t^2+b^2)}\,dt$

$$\widehat{\frac{1}{t^2+a^2}}(\xi) = \frac{\pi}{a}\exp(-a|\xi|)$$

$$\widehat{\frac{t}{t^2+a^2}}(\xi) = i\frac{d}{d\xi}\left(\frac{\pi}{a}\exp(-a|\xi|)\right) = i\frac{\pi}{a}(-a)|\xi| = \cancel{-i\pi\,\xi\,\mathrm{sgn}\,\xi}$$

$$= i\frac{\pi}{a}(-a)\,\mathrm{sgn}(\xi)e^{-a|\xi|} \quad =0$$

Plancherel

$$\int \frac{t^2\,dt}{(t^2+a^2)(t^2+b^2)} = \frac{1}{2\pi}\int i\pi\,\mathrm{sgn}\,\xi\, e^{-a|\xi|}\,\overline{\left(-i\pi\,\mathrm{sgn}\,\xi\, e^{-b|\xi|}\right)}\,d\xi$$

$$= \frac{1}{2\pi}\int (-i)\,i\pi^2\cdot 1\, e^{-(a+b)|\xi|}\,d\xi = \frac{\pi}{2}\cdot 2\cdot\frac{1}{a+b}$$

GÖ13 $\quad u'(t) + 2u(t) + \underbrace{e^{-2t}\int_{-\infty}^{t} e^{2\tau}u(\tau)\,d\tau}_{} = \delta(t)$

$$= \int_{-\infty}^{t} e^{2(\tau-t)}u(\tau)\,d\tau =$$

$$= \int_{-\infty}^{+\infty} e^{-2(t-\tau)}\chi_{\tau<t}(\tau)u(\tau)\,d\tau = \int_{-\infty}^{+\infty} e^{-2(t-\tau)}\theta(t-\tau)u(\tau)\,d\tau$$

$u'(t) + 2u(t) + u * h(t) = \delta(t)$, där $h(t) = e^{-2t}\theta(t)$

Fouriertransf. $\quad i\xi\hat{u}(\xi) + 2\hat{u}(\xi) + \hat{u}(\xi)\hat{h}(\xi) = 1$

$$\hat{h}(\xi) = \int_0^{\infty} e^{-2t}e^{-it\xi}\,dt = \frac{1}{2+i\xi}$$

Ekv. $\quad \left(i\xi + 2 + \frac{1}{2+i\xi}\right)\hat{u}(\xi) = 1$

$$\hat{u}(\xi) = \frac{2+i\xi}{(2+i\xi)^2+1} = \frac{2+i\xi}{(2+i\xi+i)(2+i\xi-i)} = \frac{2+i\xi+i+2+i\xi-i}{2(2+i\xi+i)(2+i\xi-i)}$$

$$= \frac{1}{2}\frac{1}{2-i+i\xi} + \frac{1}{2}\frac{1}{2+i+i\xi}$$

Försök med $e^{-(2\pm i)t}\theta(t)$

$$\int_0^\infty e^{-(2\pm i)t} e^{-it\xi}\,dt = \int_0^\infty e^{(-2-\xi i \mp i)t}\,dt = \left[\frac{e^{(-2-i\xi \mp i)t}}{-2-i\xi \mp i}\right]_0^\infty =$$

$$= \frac{1}{2+i\xi \pm i}$$

Alltså: $\hat{u}(\xi) = \frac{1}{2}\frac{1}{2+i\xi+i} + \frac{1}{2}\frac{1}{2+i\xi-i}$

$$u(t) = \left[\frac{1}{2}\exp(-2t-it) + \frac{1}{2}\exp(-2t+it)\right]\theta(t) =$$

$$= e^{-2t}\cos(t)\cdot\theta(t)$$

Test!

$$e^{-2t}\int_{-\infty}^{t} e^{2\tau} u(\tau)\,d\tau = e^{-2t}\int_{-\infty}^{t}\cos\tau\,\theta(\tau)\,d\tau =$$

$$= e^{-2t}\int_0^t \cos\tau\,d\tau = e^{-2t}\sin t$$

VL: $-2e^{-2t}\cos t\,\theta(t) - e^{-2t}\sin t + \delta + 2e^{-2t}\cos t\,\theta(t)$

$$+ e^{-2t}\sin t = \delta$$

7.4.1 (a) $\mathcal{F}_s(e^{-kx})(\xi) =$

$$= \int_0^\infty e^{-kx} \sin x\xi \, dx = \frac{1}{2i} \int_0^\infty e^{-kx+ix\xi} - e^{-xk-ix\xi} \, dx$$

$$= \frac{\xi}{k^2+\xi^2}$$

(b) $\mathcal{F}_c(e^{-kx}) = \dfrac{k}{k^2+\xi^2}$

EÖ 12 $f(t) = \int_0^1 \sqrt{\omega}\, e^{\omega^2} \cos \omega t \, d\omega$

$$\int_{-\infty}^{+\infty} |f'(t)|^2 \, dt = ?$$

$$f(t) = \mathcal{F}_c\left(\sqrt{\omega}\, e^{\omega^2} \chi_{(0,1)}(\omega)\right)(t)$$

$$f'(t) = -\int_0^1 \omega^{\frac{3}{2}} e^{\omega^2} \sin \omega t \, d\omega$$

$$f' = -\mathcal{F}_s\left(\omega^{\frac{3}{2}} e^{\omega^2} \chi_{(0,1)}\right)$$

Plancherel för $\mathcal{F}_s$:

$$\int_0^\infty |\mathcal{F}_s g|^2 \, d\xi = \frac{\pi}{2} \int_0^\infty |g|^2 \, dx$$

Alltså: $\displaystyle\int_0^\infty |f'(t)|^2 dt = \frac{\pi}{2}\int_0^1 \omega^3 e^{2\omega^2} d\omega =$

$$= \frac{\pi}{2}\int \frac{1}{4}\omega^2 \cdot 4\omega e^{2\omega^2} d\omega = \left\{ \begin{array}{c}\text{Partiell} \\ \text{integr.}\end{array} \right\} = \frac{\pi}{16}(e^2+1)$$

$\displaystyle\int_{-\infty}^0 |f'(t)|^2 dt = \int_0^\infty \ldots = \ldots$ ty f' udda

Alt. $\displaystyle f(t) = \frac{1}{2}\int_0^1 \sqrt{\omega}\, e^{\omega^2}(e^{i\omega t} + e^{-i\omega t})\, dt =$

$$= \frac{1}{2}\int_0^1 \sqrt{|\omega|}\, e^{\omega^2} e^{-i\omega t}\, dt$$