EÖ18 insignal $\frac{1}{4+t^2}$ ger utsignal **Mån LV5**

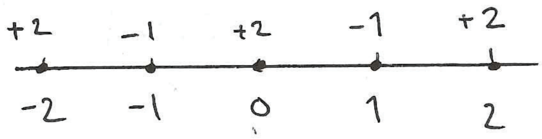
$$e^{-2t^2}$$

$$\widehat{\frac{1}{4+t^2}} = \frac{\pi}{2} e^{-2|\omega|} \qquad \widehat{e^{-2t^2}} = \sqrt{\frac{\pi}{2}}\, e^{-\frac{\omega^2}{8}}$$

systemfunktionen

$$\hat{h}(\omega) = \frac{\sqrt{\frac{\pi}{2}}\, e^{-\omega^2/8}}{\frac{\pi}{2} e^{-2|\omega|}} = \sqrt{\frac{2}{\pi}}\, e^{-\frac{\omega^2}{8}+2|\omega|}$$

$$f(t) = \sum_{-\infty}^{+\infty} \left(2\delta(t-2n) - \delta(t-2n-1)\right)$$



f är 2-periodisk → Borde ha en F-serieutveckling

Utveckla f i Fourierserie $f = \sum_{-\infty}^{+\infty} c_k e^{in\frac{\pi}{1}t}$ $\{L=1\}$

$$c_k = \frac{1}{2L}\int_{-1+\varepsilon}^{1+\varepsilon} f(t)e^{-ik\pi t}\,dt = \frac{1}{2L}\left(2-e^{-ik\pi}\right) =$$

$$= \frac{1}{2L}\left(1-(-1)^k\right)$$ $\{$obs! $L=1$ $\}$ $\{0<\varepsilon<1\}$

Insignal $e^{i\omega t}$ ger utsignal $\hat{h}(\omega)e^{i\omega t}$

f ger utsignal

$$\sum_{k=-\infty}^{+\infty} c_k \hat{h}(k\pi) e^{ik\pi t} = \sum_{-\infty}^{+\infty} \frac{1}{2}\left(2-(-1)^k\right)\sqrt{\frac{2}{\pi}}\, e^{-\frac{k^2\pi^2}{8}+2|k|\pi} e^{ik\pi t}$$

3.3.10b)

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^6}$$

$$f_{17}(\theta) = \theta(\pi-|\theta|) = \frac{8}{\pi}\sum_{n=1}^{\infty} \frac{\sin(n-1)\theta}{(2n-1)^3}$$

$\{ -\pi < \theta < \pi, \quad f \text{ udda} \}$

Parseval: $f = \sum_1^{\infty} b_n \sin n\theta$

$$\int_0^{\pi} |f(\theta)|^2 d\theta = \frac{\pi}{2}\sum_1^{\infty} |b_n|^2$$

s. 79 i Folland

$\{$ Härledn. $\displaystyle\int_0^{\pi} |f|^2 d\theta = \sum_1^{\infty} |b_n|^2 \underbrace{\int_0^{\pi} \sin^2 n\theta \, d\theta}_{\frac{\pi}{2}} \}$

$$\int_0^{\pi} |f_{17}(\theta)|^2 d\theta = \int_0^{\pi} \theta^2(\pi-|\theta|)^2 d\theta = \dots\dots = \frac{\pi^5}{30}$$

$$\frac{\pi^5}{30} = \frac{\pi}{2}\left(\frac{8}{\pi}\right)^2 \sum_1^\infty \frac{1}{(2n-1)^6}$$

$$\sum_1^\infty \frac{1}{(2n-1)^6} = \frac{\pi^6}{960}$$

3.4.7a) Bästa approx i norm av $f(x) = x$ i $[0,\pi]$ med funktioner av typ $a_0 + a_1\cos x + a_2\cos 2x$

dvs. minimera $\displaystyle\int_0^\pi |f(x) - (a_0 + a_1\cos x + a_2\cos 2x)|^2\,dx$

$$\left\{ \int_a^b |f(x) - \sum_1^N d_n\varphi_n(x)|^2\,dx \quad \text{minimeras då} \quad d_n = c_n(f) = \frac{1}{\|\varphi\|^2}\langle f, \varphi_n\rangle \right\}$$

Min. då $\displaystyle a_k = \frac{1}{\|\cos kx\|}\int_0^{+\infty} f(x)\cos kx\,dx$

Sätt $f(x) = x$ i $[-\pi,\pi]$

Läs av $a_0 = \frac{\pi}{2}$, $a_1 = -\frac{4}{\pi}$

$$f(\theta) = f_1(x) = |x| = \frac{\pi}{2} - \frac{4}{\pi}\sum_1^\infty \frac{\cos(2n-1)}{(2n-1)^2}$$

Svar: $\frac{\pi}{2} - \frac{4}{\pi}\cos x$

3.5.4

S-L-problem.

$$f'' + \lambda f = 0 \quad i \ [0, L]$$

$$f'(0) = f(L) = 0$$

$\lambda < 0$ sätt $\lambda = -\mu^2$, $\mu > 0$

$\rightarrow f(x) = a \cosh \mu x + b \sinh \mu x$ {allm. lösn.}

$f'(x) = a\mu \sinh \mu x + b\mu \sinh \mu x$

$f'(0) = 0$ ger $b\mu = 0$, $b = 0$

$\rightarrow f(x) = a \cosh \mu x$

$f(L) = 0$ ger $a = 0$ $\Longrightarrow$ Inga lösn.!

$\lambda = 0$ $f(x) = ax + b$

Får $a = 0$, $b = 0$ $\Longrightarrow$ Inga lösn.!

$\lambda > 0$ sätt $\lambda = \mu^2$, $\mu > 0$

Lösn. till ekvationen: $f(x) = a \cos \mu x + b \sin \mu x$

$f'(x) = -a\mu \sin \mu x + b\mu \cos \mu x$

$f'(0) = 0$ ger $b = 0$

$\rightarrow f(x) = a \cos \mu x$ kasta a

$f(L) = 0$ ger $\cos \mu L = 0 \rightarrow \mu L = \left(n - \frac{1}{2}\right)\pi$

$n = 1, 2, \ldots$

$$f(x) = \cos\frac{(n-\frac{1}{2})\pi x}{L}$$ är egenfunktionerna

Egenvärden: $\left((n-\frac{1}{2})\frac{\pi}{L}\right)^2$

$$\left\| \cos\frac{(n-\frac{1}{2})\pi x}{L} \right\|^2_{L^2[0,\pi]} = \int_0^L \cos^2\frac{(n-\frac{1}{2})\pi x}{L}\,dx =$$

$$= \frac{1}{2}\int_0^L \left(1 + \cos\frac{(2n-1)\pi x}{L}\right) dx = \frac{L}{2} + \frac{L}{(2n-1)\pi}\left[\sin\frac{(2n-1)\pi x}{L}\right]_0^L =$$

$$= \frac{L}{2}$$

Normaliserade egenfunktioner: $\sqrt{\frac{2}{L}}\cos\frac{(n-\frac{1}{2})\pi x}{L}$

Jfr. (3.5.3) $f'' + \lambda f = 0 \qquad f(0) = f'(L) = 0$

Egenfunktioner: $\sin\frac{(n-\frac{1}{2})\pi x}{L}$

sätt $x = L - x'$ $\longrightarrow$ $\sin\frac{(n-\frac{1}{2})\pi}{L}(L-x') = (-1)^{n+1}\cos\frac{(n-\frac{1}{2})\pi}{L}x'$

7.2.12 $f_a = \frac{1}{\pi}\frac{a}{a^2+x^2}$

Visa att $f_a * f_b = f_{a+b}$

$\hat{f}_a \hat{f}_b \longleftrightarrow \hat{f}_{a+b}$

$\hat{f}_a(\xi) = e^{-a|\xi|}$

$e^{-a|\xi|}e^{-b|\xi|} = e^{-(a+b)|\xi|}$ klart

$$f_y(x) = \frac{1}{\pi} \frac{y}{x^2+y^2}$$

Lösn: $u(x,y) = \frac{1}{\pi} \int g(x-t) \frac{y}{t^2+y^2} \, dt = f_y * g(x)$

$$f_{y_1} * f_{y_2} * g = f_{y_1+y_2} * g$$

b) $g_a = \dfrac{\sin ax}{\pi x}$ $\quad a, b > 0$

Visa $g_a * g_b = g_{\min(a,b)}$

$\hat{g}_a = \chi_{(-a,a)}$ $\quad \chi_{(-a,a)} \cdot \chi_{(-b,b)} = \chi_{(-\min(a,b), \min(a,b))}$

EÖ 11 $f(t) = \displaystyle\int_0^2 \frac{\sqrt{w}}{1+w} e^{iwt} \, dw$

$$f(t) = 2\pi \cdot \mathcal{F}^{-1}\left(\frac{\sqrt{w}}{1+w} \chi_{(0,2)} \right)$$

$$\mathcal{F} f(w) = 2\pi \cdot \frac{\sqrt{w}}{1+w} \chi_{(0,2)}(w)$$

a) $\displaystyle\int_{-\infty}^{+\infty} f(t) \cos t \, dt = \frac{1}{2} \int_{-\infty}^{+\infty} f(t) \left(e^{it} + e^{-it} \right) dt =$

$$\frac{1}{2} \left(\hat{f}(-1) + \hat{f}(1) \right) = \frac{1}{2} \left(0 + 2\pi \frac{1}{2} \right) = \frac{\pi}{2}$$

b) $\displaystyle\int_{-\infty}^{+\infty} |f(t)|^2 \, dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |\hat{f}(w)|^2 \, dw = \frac{1}{2\pi} (2\pi)^2 \int_0^2 \frac{w}{(1+w)^2} \, dw$

$$= 2\pi \int_0^2 \left(\frac{1}{1+w} - \frac{1}{(1+w)^2} \right) = 2\pi \left(\ln 3 - \frac{2}{3} \right)$$

$$f(x) = \cos\frac{\left(n - \frac{1}{2}\right) x \pi}{L}$$ är egenfunktioner

Egenvärden :