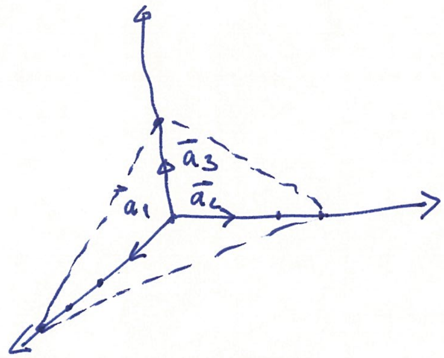
2013 08 27

Lösningar till tentauppgifter:

1

a)



$|\vec{a}_1| = |\vec{a}_2| = |\vec{a}_3| = 5$ nm

Planet i fig. 1. skär x, y, z axlar i $4a_1$, $3a_2$ och $2a_3$.

Inversa tal (i enheter av a_1, a_2, a_3)

$\frac{1}{4}, \frac{1}{3}, \frac{1}{2}$

Multiplicera med 12

Miller index: $\underline{(3,4,6)}$

b) $d_{346} = ?$

Från Fig. 1 $\Rightarrow$ $\vec{a}_1 \perp \vec{a}_2 \perp \vec{a}_3$. Vidare $a_1 = a_2 = a_3$

$\Rightarrow$ reciproka gittret spänns av 3 st vektorer $\vec{b}_i$ där

$|\vec{b}_1| = |\vec{b}_2| = |\vec{b}_3| = \frac{2\pi}{a} = 1.26$ nm^{-1} och $\vec{b}_1 \perp \vec{b}_2 \perp \vec{b}_3$

$\Rightarrow |\vec{b}_{346}| = \sqrt{3^2+4^2+6^2} \cdot \frac{2\pi}{a} = 9.81$ nm

$$d_{346} = \frac{a}{\sqrt{3^2+4^2+6^2}} = \underline{0.64 \text{ nm}}$$

2

a) $E = E_c + E_{rep} = -N\alpha \frac{e^2}{r} + \frac{1}{2} N\, 6 \cdot \frac{A}{r^{12}}$

N = antalet joner i kristallen   # n.g för NaCl strukturer

$$E_{NaCl} = N\left(-\frac{\alpha e^2}{r} + 3\frac{A}{r^{12}}\right)$$

$$\frac{\partial E}{\partial r} = 0$$

$$\frac{\partial E}{\partial r} = N\left(\frac{\alpha e^2}{r^2} - \frac{36A}{r^{13}}\right) = 0 \Rightarrow r = \left(\frac{36A}{\alpha e^2}\right)^{\frac{1}{11}}$$

$$E_{NaCl} = N\left(-\alpha e^2\left(\frac{\alpha e^2}{36A}\right)^{\frac{1}{11}} + 3A\left(\frac{\alpha e^2}{36A}\right)^{12/11}\right) = -\frac{11}{12} N\left(\alpha e^2\right)^{12/11}(36A)^{-1/11}.$$

b) $E = -N\alpha \frac{e^2}{r} + \frac{1}{2} N\, 8\, \frac{A}{r^{12}}$

n.g för CsCl strukturer

$$\frac{\partial E}{\partial r} = 0 \Rightarrow r = \left(\frac{48A}{\alpha e^2}\right)^{1/11}$$

$$E_{CsCl} = -\frac{11}{12} N\left(\alpha e^2\right)^{12/11}(48)^{-1/11}$$

c) NaCl $E = -\frac{11}{12}\frac{\alpha e^2}{r}$ $r = \left(\frac{36A}{\alpha e^2}\right)^{1/11}$

CsCl $E = -\frac{11}{12}\frac{\alpha e^2}{r}$ $r = \left(\frac{48A}{\alpha e^2}\right)^{1/11}$

NaCl : $\alpha^{12/11} \cdot 36^{-1/11} = 1.3274$

CsCl : $\alpha^{12/11} \cdot 48^{-1/11} = 1.3053$

<u>NaCl</u> strukturen har lägre energin.

3 $$\mathcal{N} = \int_{\omega_1}^{\omega_2} d\omega \; D(\omega)\langle n(\omega)\rangle = \int_{\omega_1}^{\omega_2} d\omega \; D(\omega) \frac{1}{e^{\frac{\hbar\omega}{k_BT}}-1}$$

$$D(\omega) \underset{\Uparrow}{=} \frac{V\omega^2}{2\pi^2 v^3} \qquad \omega_1 = 2\pi\cdot 4.1\cdot 10^6$$
$$\omega_2 = 2\pi\, 4\cdot 10^6$$

Debye app.

Obs $k_BT \approx 25$ meV

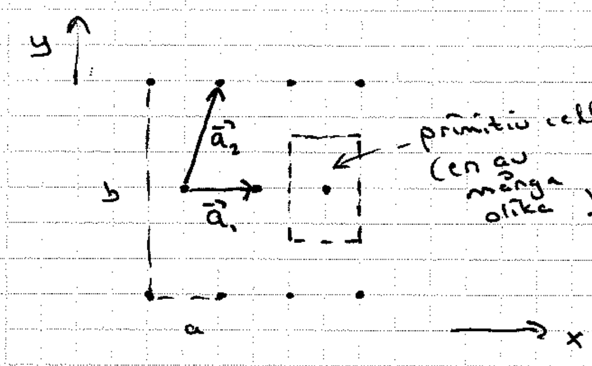
för $f = 4\cdot 10^6$ Hz $\Rightarrow \omega = 2\pi\cdot f \Rightarrow$ $\hbar\omega \ll k_BT \Rightarrow$

$$\frac{1}{e^{\hbar\omega/k_BT}-1} \approx \frac{1}{1+\frac{\hbar\omega}{k_BT}-1} \approx \frac{k_BT}{\hbar\omega}$$

$$\mathcal{N} = \frac{V}{2\pi^2 v^3}\frac{k_BT}{\hbar}\int_{\omega_1}^{\omega_2}\frac{\omega^2 d\omega}{\omega} = \frac{Vk_BT}{2\pi^2 v^3\hbar}\cdot\frac{1}{2}\left(\omega_2^2-\omega_1^2\right)$$

$$\mathcal{N} = \frac{10^{-6}\cdot 1.38\cdot 10^{-23}\cdot 300}{2\pi^2\cdot(6000)^3\cdot 1.05\cdot 10^{-34}\cdot 2}\left[4\pi^2(4.1\cdot 10^6)^2 - 4\pi^2(4\cdot 10^6)^2\right] = 1.48\cdot 10^8$$

a)



$\vec{a}_1 = a\hat{x}$

$\vec{a}_2 = \frac{a}{2}\hat{x} + \frac{b}{2}\hat{y}$

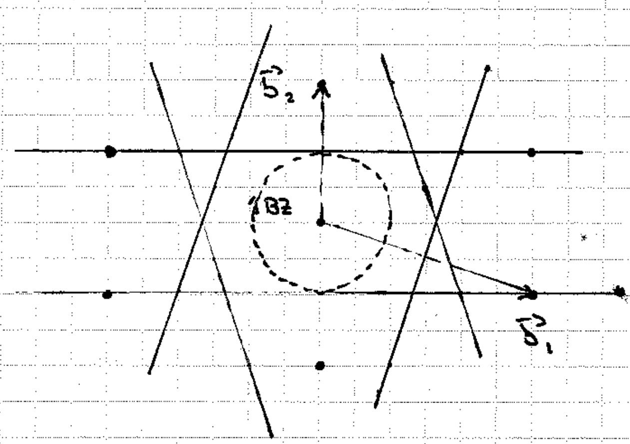
b) reciprokt gitter $\vec{b}_1, \vec{b}_2$

$$\left.\begin{aligned} \vec{b}_1 \cdot \vec{a}_1 &= 2\pi \\ \vec{b}_1 \cdot \vec{a}_2 &= 0 \end{aligned}\right\} \Rightarrow \vec{b}_1 = \frac{2\pi}{b}\left(3\hat{x} - \hat{y}\right)$$

$$\left.\begin{aligned} \vec{b}_2 \cdot \vec{a}_2 &= 2\pi \\ \vec{b}_2 \cdot \vec{a}_1 &= 0 \end{aligned}\right\} \Rightarrow \vec{b}_2 = \frac{2\pi}{b} \cdot 2\hat{y}$$

reciprokt gitter :

$\frac{2\pi}{b}$



c) cirkulär Fermiyta : $\text{volym} = \pi r^2 = \pi\left(\frac{2\pi}{b}\right)^2 = V_c$

volym av 1sta BZ : $V = |\vec{b}_1 \times \vec{b}_2| = 6\left(\frac{2\pi}{b}\right)^2$

Full 1sta BZ $\Leftrightarrow$ 2 elektroner / atom

↑ (2 från spinn)

$$n = 2 \cdot \frac{V_c}{V} = 2\,\frac{\pi}{6} = \frac{\pi}{3} \gtrsim 1$$

$$\text{antal } e^- / \text{atom} = \frac{\pi}{3} \approx 1$$

a) Drude: $\vec{j} = -ne\vec{v} = \sigma\vec{E}$

e-täthet drifthast.

$$m\dot{\vec{v}} = -e\vec{E} - m\frac{\vec{v}}{\tau}$$ ← kollisionstid

$$= -e\vec{E} - mf\vec{v} = -e\vec{E} - m(f_0 + f_{e\text{-}e} + f_{e\text{-}p})\vec{v}$$

där de olika processerna bidrar separat

jämvikt $\dot{\vec{v}} = 0 \Rightarrow \vec{v} = \frac{-e\vec{E}}{m(f_0 + f_{e\text{-}e} + f_{e\text{-}p})}$

$\therefore \sigma = \frac{ne^2}{m(f_0 + f_{e\text{-}e} + f_{e\text{-}p})}$ och resistiviteten

$$\underline{\underline{\rho}} = \frac{m}{ne^2}(f_0 + f_{e\text{-}e} + f_{e\text{-}p}) = \underline{\underline{\frac{m}{ne^2}(f_0 + aT^2 + bT^5)}}$$

dvs. termer med olika temperaturberoende

b) medelfria vägen $l = v_F\tau = \frac{v_F}{f_{e\text{-}e}} = \frac{\hbar v_F}{\hbar f_{e\text{-}e}} =$

$$\approx \hbar v_F \frac{\varepsilon_F}{(k_B T)^2} = \left[\varepsilon_F = k_B T_F\right] =$$

$$= \frac{\hbar v_F}{\varepsilon_F}\left(\frac{T_F}{T}\right)^2$$

$v_F = \sqrt{\frac{2\varepsilon_F}{m}}$

$\frac{\hbar v_F}{\varepsilon_F} = \hbar\sqrt{\frac{2}{m\varepsilon_F}} = \sqrt{\frac{2\hbar^2}{m\varepsilon_F}} = 2k_F$

$$\approx k_F\left(\frac{T_F}{T}\right)^2 \approx \left[k_F \sim \frac{\pi}{a}\right] =$$

$$= a\left(\frac{T_F}{T}\right)^2$$ $\varepsilon_F = 3\,eV \sim T_F \approx 30\,kK$

gitterkonstanten

vid rumstemp. $\left(\frac{T_F}{T}\right)^2 \sim 10^4$

$\approx 10^4 a$ om $a \sim nm$

$\Rightarrow l \sim 10\,\mu m$ långt!