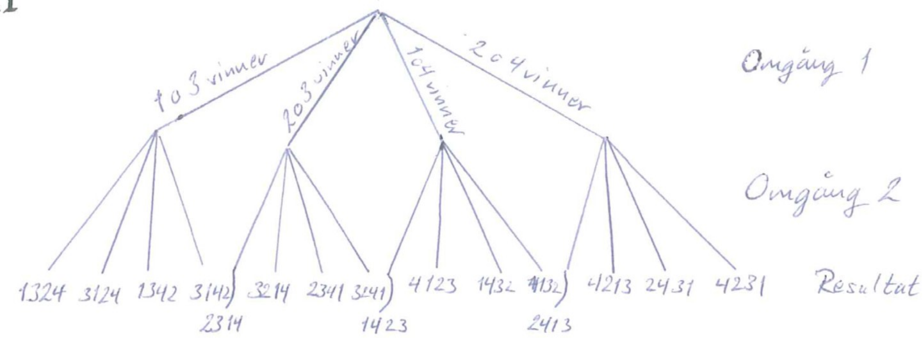
Studentskrivna Lösningar till övningsuppgifter i

Grundkurs i matematisk statistik och bioinformatik

Skrivna av Jonas Elmwall (Student på Bioteknik) 2008

Statistik & Bioinformatik – Övningsuppgifter

2.1



a) $S = \{1324, 1342, 1423, 1432, 2314, 2341, 2413, 2431, 3124, 3142, 3214, 3241, 4123, 4132, 4213, 4231\}$

b) $A = \{1324, 1342, 1432, 1423\}$

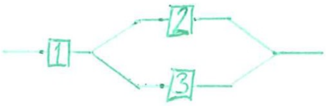
c) $B = \{2314, 2341, 2413, 2431, 3214, 3241, 4213, 4231\}$

d) $A \cup B = \{1324, 1342, 1423, 1432, 2314, 2341, 2413, 2431, 3214, 3241, 4213, 4231\}$

$A \cap B = \emptyset$, A och B disjunkta.

$A' = \{2314, 2341, 2413, 2431, 3124, 3142, 3214, 3241, 4123, 4132, 4213, 4231\}$

2.3



a) $A = \{SSF, SFS, FSS\}$

b) $B = \{SSF, SFS, FSS, SSS\}$

c) $C = \{SSF, SFS, SSS\}$

d) $C' = \{FFF, SFF, FSF, FFS, FSS\}$

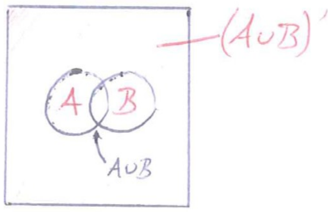
$A \cup C = \{SSF, SFS, FSS, SSS\} = B$

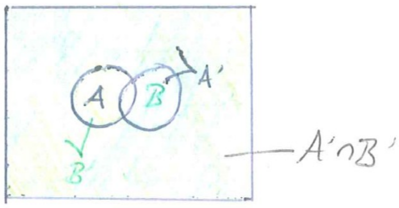
$A \cap B = \{SSF, SFS\}$

$B \cup C = \{SSF, SFS, FSS, SSS\} = B$

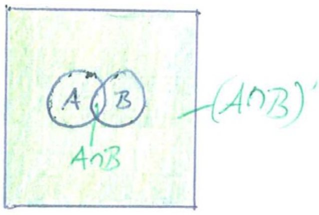
$B \cap C = \{SSF, SFS, SSS\} = C$

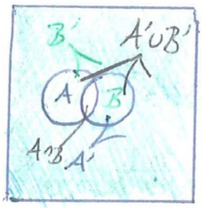
2.9 a) Verifiera $(A\cup B)' = A'\cap B'$





b) $(A\cap B)' = A'\cup B'$

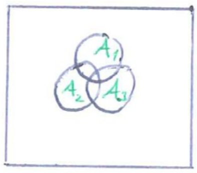




2.13 $P(A_1) = 0{,}22$; $P(A_2) = 0{,}25$; $P(A_3) = 0{,}28$

$P(A_1\cap A_2) = 0{,}11$; $P(A_1\cap A_3) = 0{,}05$; $P(A_2\cap A_3) = 0{,}07$

$P(A_1\cap A_2\cap A_3) = 0{,}01$



a) Beräkna sannolikheten för ~~P~~$(A_1\cup A_2)$ $= \underline{P(A_1\cup A_2)}$

$P(A_1\cup A_2) = P(A_1) + P(A_2) - P(A_1\cap A_2) = 0{,}22 + 0{,}25 - 0{,}11 =$

$= \underline{\underline{0{,}36}}$

b) $\underline{P(A'\cap A_2')}$

$P(A'\cap A_2') = P((A_1\cup A_2)') = 1 - P(A_1\cup A_2) = 1 - 0{,}36 = \underline{\underline{0{,}64}}$

c) $\underline{P(A_1\cup A_2\cup A_3)}$

$P(A_1\cup A_2\cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1\cap A_2) - P(A_1\cap A_3) -$

$- P(A_2\cap A_3) + P(A_1\cap A_2\cap A_3) = 0{,}22 + 0{,}25 + 0{,}28 - 0{,}11 -$

$- 0{,}05 - 0{,}07 + 0{,}01 = \underline{\underline{0{,}53}}$

d) $P(A_1'\cap A_2'\cap A_3') = P((A_1\cup A_2\cup A_3)') = 1 - P(A_1\cup A_2\cup A_3) =$

$= 1 - 0{,}53 = \underline{\underline{0{,}47}}$

Fortsättning 2.13

e) $P(A_1' \cap A_2' \cap A_3)$

$P(A_1' \cap A_2' \cap A_3) = P(A_3) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$

$= 0{,}28 - 0{,}05 - 0{,}07 + 0{,}01 = \underline{0{,}17}$

f) $P((A_1' \cap A_2') \cup A_3)$

$P((A_1' \cap A_2') \cup A_3) = 1 - P(A_1) - P(A_2) + P(A_1 \cap A_2) + P(A_1 \cap A_3) +$
$+ P(A_2 \cap A_3) - P(A_1 \cap A_2 \cap A_3) = 1 - 0{,}22 - 0{,}25 + 0{,}11 + 0{,}05 +$
$+ 0{,}07 - 0{,}01 = \underline{0{,}75}$

2.15 $P(A) = 0{,}30$; $0{,}50$

a) $P(A) + P(B) \neq 1$ då det även finns andra tjänster.

b) $P(A') = 1 - P(A) = 1 - 0{,}30 = \underline{0{,}70}$

c) $P(A \cup B) = 0{,}30 + 0{,}50 = \underline{0{,}80}$

d) $P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B) = 1 - 0{,}80 = \underline{0{,}20}$

2.29 a) 26 bokstäver i engelskan $\Rightarrow n_1 = 26$, 2 bokstäver $\Rightarrow N = n_A n_B$

$N_1 = n_1^2 = 26^2 = \underline{676}$

Med siffror: $n_2 = 26 + 10 = 36$

$\Rightarrow N_2 = n_2^2 = \underline{1296}$

b) 3 bokstäver $\Rightarrow N = n_A n_B n_C$

$\Rightarrow N_1 = 26^3 = \underline{17576}$

Med siffror: $N_2 = 36^3 = \underline{46656}$

c) 4 bokstäver

$\Rightarrow N_1 = 26^4 = \underline{456\,976}$

$N_2 = 36^4 = \underline{1\,679\,616}$

d) 97 786 ~~tagna~~ otagna $= N(A)$, 36^4 är möjliga

~~Kombinera~~ $\Rightarrow$ Tagna: $N(A') = 36^4 - 97\,786$

$P(A') = \frac{N(A')}{N} = \frac{36^4 - 97\,786}{36^4} \approx \underline{0{,}942}$

2.33 $n = 15$ spelare

a) $k = 9$ spelare som ska väljas

Facit säger 9!, vilket ej kan stämma.

Antal kombinationer: $\binom{15}{9} = \frac{n!}{k!(n-k)!} = \frac{15!}{9!\,6!} = \underline{5005}$ st

b) Med hänsyn till ordning:

Antal permutationer: $P_{9,15} = \frac{n!}{(n-k)!} = \frac{15!}{6!} = \underline{1\,816\,214\,400}$

c) $n_v = 5$ vänsterhänta; $n_H = 10$ högerhänta

$k_v = 3$ $\qquad$ $k_H = 6$

Vänsterh: $N_v = \binom{5}{3}$; Högerh: $N_H = \binom{10}{6}$

$$N = N_v N_H = \binom{5}{3}\binom{10}{6} = \frac{n_v!}{k_v!(n_v-k_v)!}\cdot\frac{n_H!}{k_H!(n_H-k_H)!} = \frac{5!}{3!\,2!}\cdot\frac{10!}{6!\,4!} =$$

$$= 10\cdot 210 = \underline{2100}$$

2.35 $n_T = 3$; $n_P = 4$; $n_K = 5$

a) $N = n_T n_P n_K = 3\cdot 4\cdot 5 = \underline{60 \text{ st}}$

b) $n_T = 1$; $n_P = 2$

$\Rightarrow N = 1\cdot 2\cdot 5 = \underline{10 \text{ st}}$

c)

2.37 A: 20, B: 15, C: 10, $k = 6$

a) Alla från skift A = A_1

$$P(A_1) = \frac{\binom{20}{6}\binom{25}{0}}{\binom{45}{6}} = \frac{\frac{20!}{\cancel{6!}\,14!}\cdot 1}{\frac{45!}{\cancel{6!}\,39!}} = \frac{20!\,39!}{14!\,45!} \approx \underline{0{,}0048}$$

b) Alla från samma = S; alla fr. B = B_1; alla fr. C = C_1

$$P(S) = P(A_1)+P(B_1)+P(C_1) - P(A_1\cancel{\cap}B_1) - P(A_1\cancel{\cap}C_1) - P(B_1\cancel{\cap}C_1) +$$

$$+ P(A_1\cap \cancel{B_1}\cap C_1) =$$

FORTSÄTTNING 2.37

$$= \frac{\binom{20}{6}+\binom{15}{6}+\binom{10}{6}}{\binom{45}{6}} = \frac{\frac{20!}{6!14!}+\frac{15!}{6!9!}+\frac{10!}{6!4!}}{\frac{45!}{6!39!}} \approx \underline{0{,}0054}$$

c) Från minst två skift = T

$$P(T) = P(S') = 1 - P(S) = 1 - 0{,}0054 = \underline{0{,}9946}$$

d) Minst ett skift ej representerat = E

A_2 = ingen fr. A; B_2 = ingen fr. B; C_2 = ingen fr. C

$$P(A_2) = \frac{\overset{=1}{\binom{20}{0}}\binom{25}{6}}{\binom{45}{6}};\quad P(B_2) = \dots;\quad P(C_2) = \dots$$

$$P(E) = P(A_2) + P(B_2) + P(C_2) - P(A_2 \cap B_2)^* - P(A_2 \cap C_2)^* -$$
$$- P(B_2 \cap C_2)^* + \cancel{P(A_2 \cap B_2 \cap C_2)}$$

[* = alla fr. samma = P(S)]

$$= \frac{\binom{25}{6}+\binom{30}{6}+\binom{35}{6} - \left(\binom{20}{6}+\binom{15}{6}+\binom{10}{6}\right)}{\binom{45}{6}} =$$

$$= \frac{\frac{25!}{19!}+\frac{30!}{24!}+\frac{35!}{29!}}{\frac{45!}{39!}} - 0{,}0054 \approx \underline{0{,}2885}$$

2.39 $n_M = 4;\ n_F = 4;\ n_{tot} = 8$

a) Minst en kvinna bland tre första = A; A' = ingen kvinna

$$P(A) = 1 - P(A') = \left[P(A') = \frac{\binom{4}{0}\binom{4}{3}}{\binom{8}{3}}\right] = 1 - \frac{1 \cdot \frac{4!}{3!}}{\frac{8!}{3!5!}} \approx \underline{0{,}929}$$

b) Alla kvinnor bland de fem första = B

$$P(B) = \frac{\binom{4}{4}\binom{4}{1}}{\binom{8}{5}} = \frac{1 \cdot \frac{4!}{1!3!}}{\frac{8!}{5!3!}} = \underline{0{,}0714}$$

c) C = annan ordning; C' = samma ordn.

$$P(C') = \frac{N(C')}{N} = [N(C') = 1;\ N = 8!] = \frac{1}{8!}$$

$$\Longrightarrow P(C) = 1 - \frac{1}{8!} = \underline{0{,}99997520}$$

2.45 a) $P(A) = 0{,}106 + 0{,}141 + 0{,}200 = \underline{\underline{0{,}447}}$

$P(C) = 0{,}215 + 0{,}200 + 0{,}065 + 0{,}020 = \underline{\underline{0{,}500}}$

$P(A \cap C) = \underline{\underline{0{,}200}}$

b) $P(A|C) = \dfrac{P(A \cap C)}{P(C)} = \underline{\underline{0{,}400}}$ = Sannolikheten för att man har blodgr. A, förutsatt att man är i etnisk gr. 3.

$P(C|A) = \dfrac{P(A \cap C)}{P(A)} \approx \underline{\underline{0{,}447}}$ = Sannolikheten för att man är i etn. gr. 3 förutsatt att man har blodgr. A.

c) $P(B) = 0{,}091 \Rightarrow P(B') = 0{,}909$

$P(B' \cap E_1) = 0{,}082 + 0{,}106 + 0{,}004 = 0{,}192$

$P(B'|E_1) = \dfrac{P(B' \cap E_1)}{P(B')} = \dfrac{0{,}192}{0{,}909} \approx \underline{\underline{0{,}211}}$

2.49 $P(A) = 0{,}6;\ P(B) = 0{,}05$

B i utfallsrum för $A \Rightarrow P(A \cap B) = P(B) = 0{,}05$

$P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{P(B)}{P(A)} = \dfrac{0{,}05}{0{,}6} \approx \underline{\underline{0{,}083}}$

2.51 $n_{I,1} = 10;\ n_{II,1} = 10;\ n_{RI} = 6;\ n_{GI} = 4;\ n_{RII} = 7;\ n_{GII} = 3$

a) $P(R_I) = \dfrac{n_{RI}}{n_{I,1}} = \dfrac{6}{10} = 0{,}6;\quad P(R_{II,2}) = \dfrac{n_{RII}+1}{n_{II,1}+1} = \dfrac{8}{11}$

$\Rightarrow P(R \text{ samma}) = 0{,}6 \cdot \dfrac{8}{11} \approx \underline{\underline{0{,}436}}$

Alternativt: $P(A \cap B) = \underbrace{P(B|A)}_{=\frac{8}{11}}\underbrace{P(A)}_{=\frac{6}{10}} = 0{,}436$

b) $P(G \text{ samma}) = 0{,}4 \cdot \dfrac{3+1}{10+1} \approx 0{,}145$

$P(\Sigma(R,G) \text{ samma}) = 0{,}436 + 0{,}145 = \underline{\underline{0{,}581}}$

Alternativt: $P(A \cap B) + P(C \cap D) = \dfrac{8}{11} \cdot \dfrac{6}{10} + \dfrac{4}{11} \cdot \dfrac{4}{10}$

2.54 $P(A_1) = 0{,}22;\ P(A_2) = 0{,}25;\ P(A_3) = 0{,}28;\ P(A_1 \cap A_2) = 0{,}11$

$P(A_1 \cap A_3) = 0{,}05;\ P(A_2 \cap A_3) = 0{,}07;\ P(A_1 \cap A_2 \cap A_3) = 0{,}01$

a) $P(A_2|A_1) = \dfrac{P(A_1 \cap A_2)}{P(A_1)} = \dfrac{0{,}11}{0{,}22} = 0{,}50$

b) $P(A_2 \cap A_3|A_1) = \dfrac{P(A_1 \cap A_2 \cap A_3)}{P(A_1)} = 0{,}01/0{,}22 = 0{,}045$

c) $P(A_2 \cup A_3|A_1) = \dfrac{P(A_1 \cap A_2) + P(A_1 \cap A_3) - P(A_1 \cap A_2 \cap A_3)}{P(A_1)} \approx 0{,}68$

Saknar facit