5.31

$$f(x,y)=\begin{cases}24xy & 0\le x\le 1,\ 0\le y\le 1,\ x+y\le 1\\ 0 & \text{Annars}\end{cases}$$

$$f_x(x)=\begin{cases}12x(1-x)^2 & 0\le x\le 1\\ 0 & \text{Annars}\end{cases} \quad (1)$$

$$\rho_{x,y}=\frac{Cov(X,Y)}{\sigma_x\sigma_y}$$

$$Cov(X,Y)=E(X,Y)-\mu_x\mu_y$$

$$E(X,Y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}xy\,f(x,y)\,dx\,dy=\int_0^1\int_0^{1-x}24x^2y^2\,dy\,dx=8\int_0^1 x^2(1-x)^3dx=\frac{2}{15}$$

$$\mu_x=\int_0^1 x f_x(x)\,dx=\int_0^1 12x^2y\,dx=\frac{2}{5}$$

$$f_y(y)=[(1)\text{ ersätt } x \text{ med } y]=12y(1-y)^2\Rightarrow\mu_y=\frac{2}{5}$$

$$\Rightarrow Cov(X,Y)=-\frac{2}{75}$$

$$\sigma_x^2=\int_0^1(x-\mu_x)^2f_x(x)\,dx=12\int_0^1(x-\mu_x)^2x(1-x)^2dx=12\int_0^1 x(x^2-2\mu_x x+\mu_x^2)(1-2x+x^2)dx=$$

$$=12\int_0^1 x(x^2-2x^3+x^4-2\mu_x x+4\mu_x x^2-2\mu_x x^3+\mu_x^2-2\mu_x^2x+\mu_x^2x^2)dx=$$

$$=12\int_0^1\left(x^5-2(1+\mu_x)x^4+(1+4\mu_x+\mu_x^2)x^3-2(\mu_x+\mu_x^2)x^2+\mu_x^2x\right)dx=$$

$$=12\left[\frac{x^6}{6}-\frac{2}{5}(1+\mu_x)x^5+(1+4\mu_x+\mu_x^2)\frac{x^4}{4}-\frac{2}{3}(\mu_x+\mu_x^2)x^3+\frac{\mu_x^2}{2}x^2\right]_0^1=$$

$$=0{,}04=\sigma_y^2\quad\Rightarrow\quad\sigma_x=\sigma_y=0{,}2$$

$$\Rightarrow\rho_{x,y}=-\frac{2}{75\cdot 0{,}2^2}=-\frac{2}{3}$$

5.35

$$E[aX+b]=aE[X]+b$$

$$Cov[X,Y]=E[XY]-E[X]E[Y]$$

$$Cov[aX+b,cY+d]=E[(aX+b)(cY+d)]-E[aX+b]E[cY+d]=$$

$$=E[acXY+adX+bcY+bd]-(aE[X]+b)(cE[Y]+d)=$$

$$=acE[XY]+adE[X]+bcE[Y]+bd-(acE[X]E[Y]+adE[X]+$$

$$+bcE[Y]+bd)=acE[XY]-acE[X]E[Y]=ac\,Cov(X,Y)$$

**5.48** $X_1, X_2, \ldots, X_n, \quad n = 100$

a) $\mu = 50; \ \sigma = 1$

$\mu_{\bar{x}} = \mu, \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1}{10} = 0{,}1$

$P(49{,}9 \le \bar{X} \le 50{,}1) = \left(\frac{49{,}9-50}{0{,}1} \le Z \le \frac{50{,}1-50}{0{,}1}\right) = (-1 \le Z \le Z) =$

$= \phi(1) - \phi(-1) = 0{,}8413 - 0{,}1587 = \underline{0{,}6826}$

b) $\mu = 49{,}8$

$P(49{,}9 \le \bar{X} \le 50{,}1) = \left(\frac{49{,}9-49{,}8}{0{,}1} \le Z \le \frac{50{,}1-49{,}8}{0{,}1}\right) = \phi(3) - \phi(1) =$

$= 0{,}9987 - 0{,}8413 = \underline{0{,}1574}$

**5.49** $n = 40; \ \mu_i = 6\text{ min}; \ \sigma = 6\text{ min}$

a) $\mu_{\bar{x}} = 6 \cdot 40 = 240\text{ min}; \quad \bar{x} = 250\text{ min}; \quad \sigma_{\bar{x}} = \frac{240}{\sqrt{40}}$ Når prog. börjar

$P(\bar{x} \le 250) = P\left(Z \le \frac{250-\mu_{\bar{x}}}{\sigma_{\bar{x}}}\right) = \phi(0{,}26) = \underline{0{,}6062}$

b)

$P(\bar{x} \le 260) = P\left(Z \le \frac{260-\mu_{\bar{x}}}{\sigma_{\bar{x}}}\right) = \phi(0{,}53) = 0{,}7019$

$P(\bar{x} > 260) = 1 - P(X \le 260) = \underline{0{,}2981}$

**5.59** $\mu_{\bar{x}} = 105 = \mu_x; \quad \sigma_x = 8; \quad n_x = 40$

$\mu_{\bar{y}} = 100 = \mu_y; \quad \sigma_y = 6; \quad n_y = 35$

a) $\bar{X}$: Approximativt normal med $\mu_{\bar{x}} = \underline{105}, \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{8}{\sqrt{40}} \approx \underline{1{,}2649}$

$\bar{Y}$: — " — $\mu_{\bar{y}} = \underline{100}, \quad \sigma_{\bar{y}} = \frac{6}{\sqrt{35}} \approx \underline{1{,}0142}$

b) $\bar{X} - \bar{Y}$: Approximativt normal med:

$E[\bar{x} - \bar{y}] = E[\bar{x}] - E[\bar{y}] = 105 - 100 = \underline{5} = \mu_{\bar{x}-\bar{y}}$

$\sigma_{\bar{x}-\bar{y}} = \sqrt{V[\bar{x}-\bar{y}]} = \sqrt{\sigma_{\bar{x}}^2 + \sigma_{\bar{y}}^2} = \underline{1{,}6213}$

c)

$P(-1 \le \bar{x} - \bar{y} \le 1) = P\left(\frac{-1-\mu_{\bar{x}-\bar{y}}}{\sigma_{\bar{x}-\bar{y}}} \le Z \le \frac{1-\mu_{\bar{x}-\bar{y}}}{\sigma_{\bar{x}-\bar{y}}}\right) = P(-3{,}70 \le Z \le -2{,}47) =$

$= \phi(-2{,}47) - \phi(-3{,}70) = 0{,}0068 - 0 = \underline{0{,}0068}$

d) $P(\bar{x} - \bar{y} \ge 10) = 1 - P(\bar{x} - \bar{y} \le 10) = 1 - P\left(Z \le \frac{10-5}{1{,}6213}\right) = 1 - \phi(3{,}08) =$

$= 1 - 0{,}999 = \underline{0{,}001}$

JA

5.60 $X_1, X_2, \dots, X_5$ $\mu_1 = \mu_2 = 20$; $\mu_3 = \mu_4 = \mu_5 = 21$

$\sigma_{1,2}^2 = 4$; $\sigma_{3,4,5}^2 = 3{,}5$

$$Y = \frac{X_1 + X_2}{2} - \frac{X_3 + X_4 + X_5}{3}$$

$$\mu_Y = 2 \cdot \frac{20}{2} - 3 \cdot \frac{21}{3} = -1; \quad \sigma_Y^2 = 2 \cdot \frac{4}{2^2} - 3\frac{3{,}5}{3^2} = \frac{5}{6} \Rightarrow \sigma_Y = \sqrt{\frac{5}{6}}$$

$$P(0 \le Y) = 1 - P(Y \le 0) = 1 - P\left(Z \le \frac{0-(-1)}{\sqrt{\frac{5}{6}}}\right) = 1 - \phi(1{,}095) =$$

$$= 1 - 0{,}8643 = \underline{\underline{0{,}1357}}$$

5.85 $\mu$ = True pH; $\sigma = 0{,}1$;

$\bar{X} = (pH_1 + pH_2 + \dots + pH_n)/n$

$$P(\mu - 0{,}02 \le \bar{X} \le \mu + 0{,}02) = 0{,}95 \Rightarrow P\left(-\frac{0{,}02}{0{,}1/\sqrt{n}} \le Z \le \frac{0{,}02}{0{,}1/\sqrt{n}}\right) = 0{,}95$$

$$\Rightarrow \phi(0{,}2\sqrt{n}) - \phi(-0{,}2\sqrt{n}) = 0{,}95 \Rightarrow 2\phi(0{,}2\sqrt{n}) - 1 = 0{,}95$$

$$\Rightarrow \phi(0{,}2\sqrt{n}) = 0{,}975$$

$$\phi(1{,}96) = 0{,}975 \Rightarrow n = 96{,}04 \Rightarrow \underline{\underline{97\ st}}$$

6.1 a) Punktskatta $\bar{X}$: $\bar{x} = \frac{\sum x_i}{n} = \frac{219{,}8}{27} \approx \underline{\underline{8{,}14}} = \hat{\mu}$

c) $\hat{\sigma}^2 = S^2 = \frac{\sum(X_i - \bar{X})^2}{n-1} = \frac{\sum X_i^2 - \frac{(\sum X_i)^2}{n}}{n-1} = \frac{1860{,}94 - \frac{219{,}8^2}{27}}{26}$

$\Rightarrow \hat{\sigma} \approx \underline{\underline{1{,}66}}$

d) $X > 10$: $\frac{4}{27} = \underline{\underline{0{,}148}}$

e) $\frac{\sigma}{\mu} = \frac{S}{\bar{x}} = \frac{1{,}66}{8{,}14} \approx \underline{\underline{0{,}204}}$

6.22
$$f(x;\theta) = \begin{cases} (\theta+1)x^\theta & 0 \le x \le 1 \\ 0 & \text{Annars} \end{cases}, \quad -1 < \theta$$

a) $E[X_i] = \mu(\theta)$, endimensionellt $\theta \Rightarrow E[X_i] = \mu(\theta)$

$$\mu(\theta) = \int_{-\infty}^{\infty} x f(x;\theta)\,dx = \int_0^1 (\theta+1)x^{\theta+1}\,dx = \left[\frac{(\theta+1)x^{\theta+2}}{\theta+2}\right]_0^1 = \frac{\theta+1}{\theta+2}$$

$$\bar{X} = \frac{\hat{\theta}+1}{\hat{\theta}+2} \Rightarrow \hat{\theta}\bar{x} + 2\bar{x} = \hat{\theta} + 1 \Rightarrow \hat{\theta} = \underline{\underline{\frac{2\bar{X}-1}{1-\bar{X}}}}$$

Lösningar skrivna av Jonas Elmwall
Lp2 2008

FORTS. 6.22 a)

$$\bar{X} = \frac{\sum x_i}{n} = 0{,}8 \Rightarrow \hat{\theta} = \frac{2 \cdot 0{,}8 - 1}{1 - 0{,}8} = \underline{3}$$

6.13

$$f(x;\theta) = 0{,}5(1+\theta x) \qquad -1 \le x \le 1, \quad -1 \le \theta \le 1$$

$$\mu(\theta) = \int_{-\infty}^{\infty} x f(x;\theta)\,dx = 0{,}5\int_{-1}^{1} x + \theta x^2\,dx = 0{,}5\left[\frac{x^2}{2} + \theta\frac{x^3}{3}\right]_{-1}^{1} = 0{,}5\left(\frac{1}{2} + \frac{\theta}{3} - \left(\frac{1}{2} - \frac{\theta}{3}\right)\right) =$$

$$= \frac{\theta}{3}$$

$$\mu(\hat{\theta}) = \frac{\hat{\theta}}{3} = \bar{x} \Rightarrow \underline{\hat{\theta} = 3\bar{x}}$$

6.9 $\quad n = 150$

a) $E(\bar{X}) = \hat{\lambda} = \sum x_i / n = \dfrac{0 \cdot 18 + 1 \cdot 37 + \ldots + 7 \cdot 1}{150} \approx 0{,}211 = \hat{\mu}$

b) $\sigma_x^2 = \lambda = \bar{x}$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \Rightarrow \hat{\sigma}_{\bar{x}} = s_{\bar{x}} = \frac{s}{\sqrt{n}}, \quad s^2 = \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1} = \frac{(0 + 1^2 \cdot 37 + \ldots + 7^2 \cdot 1) + \frac{(37 + \ldots + 7)^2}{150}}{149}$$

$$\Rightarrow \hat{\sigma}_{\bar{x}} = \underline{0{,}118}$$

6.4 $\quad n = 20$

a) $X$ = Personer som äger TI $= 10$, $\quad \hat{p} = \dfrac{X}{n} = \underline{0{,}5}$

6.22

$$f(x;\theta) = \begin{cases} (\theta+1)x^{\theta} & 0 \le x \le 1 \\ 0 & \text{Annars} \end{cases}, \quad n = 10$$

b) $L(\theta) = \prod_{i=1}^{10} (\theta+1)x_i^{\theta} = (\theta+1)^{10} \prod_1^{10} x_i^{\theta}$

$$\ell(\theta) = 10\ln(\theta+1) + \theta\sum_1^{10} \ln x_i$$

$$\ell'(\theta) = \frac{10}{\theta+1} + \sum \ln x_i, \quad \underbrace{\ell'(\hat{\theta})}_{=0} \Rightarrow \frac{10}{\hat{\theta}+1} - \sum_1^{10} \ln x_i = 0 \Rightarrow \hat{\theta} = \frac{10}{-\sum \ln x_i} - 1 \approx \underline{3{,}116}$$

6.20 $\quad X \sim Bin(n,p); \; n = 20; \; x = 3$

a) $b(x;n,p) = \binom{n}{x} p^x (1-p)^{n-x}$

$$L(p) = \prod_{i=1}^{n} \binom{n}{x_i} p^{x_i}(1-p)^{n-x_i} = \left(\binom{n}{x} p^x (1-p)^{n-x}\right)^{20}$$

$$\ell(p) = 20\ln\left(\binom{n}{x} p^x (1-p)^{n-x}\right) = 20\left(\ln\binom{n}{x} + x\ln p + (n-x)\ln(1-p)\right)$$

$$\ell'(p) = 20\left(\frac{x}{p} - \frac{n-x}{1-p}\right), \quad \ell'(\hat{p}) = 0 \Rightarrow \frac{20x}{\hat{p}} = 20\frac{n-x}{1-\hat{p}} \Rightarrow \underline{\hat{p} = \frac{x}{n}}$$

6.20b) $E(\hat{p}) = E\left(\frac{X}{n}\right) = \frac{1}{n}E(X) = \frac{1}{n}(np) = p \Rightarrow$ Ja

6.24 $f(x;\theta) = \frac{x}{\theta}e^{-\frac{x^2}{2\theta}}, \quad x > 0, \quad x_1, x_2, \ldots, x_n$

a) $L(\theta) = \prod_{i=1}^{n}\left(\frac{x_i}{\theta}e^{-\frac{x_i^2}{2\theta}}\right) = \left(\frac{1}{\theta}\right)^n \prod_1^n x_i \cdot e^{-\frac{\sum x_i^2}{2\theta}}$

$l(\theta) = -n\ln\theta + \sum_1^n x_i - \frac{\sum_1^{10} x_i^2}{2\theta}$

$l'(\theta) = -\frac{n}{\theta} + \frac{\sum_1^{10} x_i^2}{2\theta^2}, \quad l'(\hat{\theta}) = 0 \Rightarrow \hat{\theta} = \frac{\sum x_i^2}{2n} = 74{,}505$

b) $F(\tilde{\mu}) = 0{,}5$

$F(x) = \int_0^x \frac{y}{\theta}e^{-\frac{y^2}{2\theta}}dy = \frac{1}{2\theta}\int_0^x ye^{-\frac{y^2}{2\theta}}dy = \begin{bmatrix} u = y^2 \\ du = 2ydy \end{bmatrix} =$

$= \frac{1}{2\theta}\int_0^{x^2} e^{-\frac{u}{2\theta}}du = \frac{1}{2\theta}(-2\theta)\left[e^{-\frac{u}{2\theta}}\right]_0^{x^2} = \left[-e^{-\frac{y^2}{2\theta}}\right]_0^x = -e^{-\frac{x^2}{2\theta}} + 1$

$F(\tilde{\mu}) = -e^{-\frac{\tilde{\mu}^2}{2\theta}} + 1 = 0{,}5 \Rightarrow e^{-\frac{\tilde{\mu}^2}{2\theta}} = 0{,}5 \Rightarrow -\frac{\tilde{\mu}^2}{2\theta} = -\ln 2$

$\Rightarrow \tilde{\mu} = \sqrt{2\theta\ln 2} \approx 10{,}16$

7.3 a) Smalare, täcker ett smalare område ⇒ Större felrisk

⋮ ork

7.6 $\sigma = 100$

a) $n = 25; \ \bar{x} = 8439 = \mu, \quad \alpha = 0{,}10 \Rightarrow 1 - \alpha = 0{,}90$

$\mu^* = \bar{X}$, punktskattningsvariabel, $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

Hjälpvariabel: $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$

$P\left(-a < \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < a\right) = 0{,}90, \quad 1 - \frac{\alpha}{2} = 0{,}95 \Rightarrow a = 1{,}65$

$P\left(-1{,}65 < \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < 1{,}65\right) = 0{,}90 \Rightarrow P\left(\bar{X} - 1{,}65\frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + 1{,}65\frac{\sigma}{\sqrt{n}}\right) = 0{,}90$

$\rightarrow I_\mu = \left(\bar{x} - \frac{1{,}65 \cdot 100}{5}, \bar{x} + \frac{1{,}65 \cdot 100}{5}\right) = (8406, 8472)$

b) $\alpha = 0{,}08, \ 1 - \frac{\alpha}{2} = 0{,}96 \Rightarrow a = 1{,}75 \Rightarrow I_\mu = (8404, 8474)$

7.33 $n = 17$

a) $\bar{x} = \frac{\sum x_i}{n} = 438$

$s^2 = \hat{\sigma}^2 = \frac{\sum x_i^2 - (\sum x_i)^2/n}{n-1} = 229 \Rightarrow \hat{\sigma} = s \approx 15$

$I_\mu = \left(\bar{x} - t_{\frac{\alpha}{2}, n-1} \frac{s}{\sqrt{n}}, \ \bar{x} + t_{\frac{\alpha}{2}, n-1} \frac{s}{\sqrt{n}}\right)$

$\nu = n - 1 = 16, \ 1 - \alpha = 0{,}95 \Rightarrow \frac{\alpha}{2} = 0{,}025 \Rightarrow$ Fr tbl A.5: $t_{\frac{\alpha}{2}, n-1} = 2{,}12$

$\longrightarrow I_\mu = (430{,}3;\ 446{,}7)$

$\Longrightarrow$ 440 ok, 450 ej ok

7.34 $n = 14;\ \hat{\sigma} = 0{,}79 = s;\ \bar{x} = 8{,}48$

a) $\nu = n - 1 = 13;\ 1 - \alpha = 0{,}95 \Rightarrow \alpha = 0{,}025 \Rightarrow$ Fr tbl A.5:
$t_{\alpha, n-1} = 1{,}771$

$\Rightarrow \bar{x} - t_{\alpha, n-1} \frac{s}{\sqrt{n}} = 8{,}11 \Rightarrow I_\mu = (8{,}11;\ \infty)$

b) $\bar{x} - t_{\alpha, n-1}\, s\sqrt{1 + \frac{1}{n}} \approx (7{,}03, \infty)$

9.41 $n = 9$

Vit: $X \sim N(\mu, \sigma_x^2)$, Svart: $Y \sim N(\mu + \Delta_0, \sigma_y^2)$

$Z = Y - X \sim N(\Delta_0, \sigma_y^2 + \sigma_x^2)$

$\bar{Z} = \bar{Y} - \bar{X} = \frac{\sum y_i - \sum x_i}{n} = 7{,}6$

$s_Z = \frac{\sum (y_i - x_i)^2 - \frac{(\sum (y_i - x_i))^2}{n}}{n-1} \approx 4{,}18, \quad 1 - \alpha = 0{,}95$

$U = \frac{\bar{Z} - \Delta_0}{s/\sqrt{n}}, \quad P(-a \leq U \leq a) = 0{,}95$

$1 - \frac{\alpha}{2} = 0{,}975, \quad \frac{\alpha}{2} = 0{,}025, \quad \nu = n - 1 = 8 \Rightarrow a = 2{,}306$

$I_\Delta = \left(\bar{z} \pm a \frac{s}{\sqrt{n}}\right) = (4{,}39;\ 10{,}81)$

$P(U \leq a) = 0{,}95 \Rightarrow a = 1{,}86 \Rightarrow I_\Delta = \left(\bar{z} - a \frac{s}{\sqrt{n}}, \infty\right) = (5{,}01;\ \infty)$

**7.20** $1-\alpha = 0{,}99;\ n = 4722;\ \hat{p} = 0{,}15;\ \hat{q} = 0{,}85$

$P\left(z_{\frac{\alpha}{2}} < \frac{\hat{p}-p}{\sqrt{p(1-p)/n}} < z_{\frac{\alpha}{2}}\right) = 0{,}99;\ 1-\frac{\alpha}{2} = 0{,}995,\ \frac{\alpha}{2} = 0{,}005$

$\nu = \infty \Rightarrow z_{\frac{\alpha}{2}} = 2{,}576$

$p = \hat{p} \pm z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}} = (0{,}1366;\ 0{,}1634)$

**7.45** $1-\alpha = 0{,}99 \Rightarrow \frac{\alpha}{2} = 0{,}005;\ n = 22 \Rightarrow \nu = 21$

$X \sim N(\mu, \sigma^2)$

$P\left(\chi_{1-\frac{\alpha}{2},\nu} < \frac{(n-1)s^2}{\sigma^2} < \chi^2_{\frac{\alpha}{2},\nu}\right) = 0{,}99;\ \chi_{1-\frac{\alpha}{2}\nu} = 8{,}033 = a;\ \chi_{\frac{\alpha}{2}\nu} = 41{,}399 = b$

$P\left(\sqrt{\frac{n-1}{b}s^2} < \sigma < \sqrt{\frac{n-1}{a}s^2}\right) = 0{,}99 \Rightarrow I_\sigma = (3{,}6;\ 8{,}1)$

**9.5** $m = 10;\ \bar{x} = 0{,}64;\ n = 10;\ \bar{y} = 2{,}05;\ X: \sigma_1 = 0{,}2;\ Y: \sigma_2 = 0{,}4$

**a)** $H_0: \mu_1 - \mu_2 = -1{,}0 = \Delta_0;\ H_a = \mu_1 - \mu_2 < -1{,}0;\ \alpha = 0{,}01$

$z = \frac{\bar{x}-\bar{y}-\Delta_0}{\sqrt{\frac{\sigma_1^2}{m}+\frac{\sigma_2^2}{n}}} = -2{,}90$

$H_a: \mu_1 - \mu_2 < \Delta_0 \Rightarrow$ Lower-tailed $\Rightarrow$ Rejection region: $z \leq -z_\alpha$

$z_{0{,}01} = 2{,}33 \Rightarrow z \leq -z_\alpha \Rightarrow$ Förkasta $H_0$!

**b)** P-value for lower-tailed test: $\phi(z) = \phi(-2{,}90) = 0{,}0019$

**c)** $\mu_1 - \mu_2 = -1{,}2 = \Delta'$

$H_a: \mu_1 - \mu_2 < \Delta_0 \Rightarrow \beta(\Delta') = 1 - \phi\left(-z_\alpha - \frac{\Delta' - \Delta_0}{\sigma}\right)$

$\sigma = \sigma_{\bar{x}-\bar{y}} = \sqrt{\frac{\sigma_1^2}{m}+\frac{\sigma_2^2}{n}} \Rightarrow \beta(\Delta') = 1 - \phi(-0{,}92) = 0{,}8212$

**d)** $m = n;\ \beta = 0{,}1;\ \mu_1 - \mu_2 = -1{,}2 = \Delta'$

$\beta = 1 - \phi\left(-z_\alpha - \frac{\Delta'-\Delta_0}{\sigma}\right) = 0{,}1 \Rightarrow \phi\left(-z_\alpha - \frac{\Delta'-\Delta_0}{\sigma}\right) = 0{,}90$

$\Rightarrow -z_\alpha - \frac{\Delta'-\Delta_0}{\sigma} = 1{,}28 \Rightarrow \frac{\Delta'-\Delta_0}{\sqrt{\frac{\sigma_1^2+\sigma_2^2}{m}}} = -3{,}61 \Rightarrow \frac{\sigma_1^2+\sigma_2^2}{m} = \left(-\frac{\Delta'-\Delta_0}{3{,}61}\right)^2$

$\Rightarrow m = 65$

9.11 ?

9.19 $\alpha = 0{,}01$; $H_0: \mu_1 - \mu_2 = \underset{=\Delta_0}{-10}$; $H_a: \mu_1 - \mu_2 < -10$

$m = 6$; $\bar{x} = 115{,}7$; $s_1 = 5{,}03$; $n = 6$; $\bar{y} = 129{,}3$; $s_2 = 5{,}38$

$H_a: \mu_1 - \mu_2 < \Delta_0 \Rightarrow$ Rejection region: $t \leq -t_{\alpha,\nu}$

$$t = \frac{\bar{x} - \bar{y} - \Delta_0}{\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{m}}} = -1{,}20$$

$$\nu = \frac{\left(\frac{s_1^2}{m} + \frac{s_2^2}{n}\right)^2}{\frac{(s_1^2/m)^2}{m-1} + \frac{(s_2^2/n)^2}{n-1}} = 9{,}96 \Rightarrow \text{Använd } \nu$$

$\Rightarrow t_{\alpha,\nu} = 2{,}821 \Rightarrow t \not\leq -t_{\alpha,\nu} \rightarrow$ Förkasta ej $H_0$!

9.21 $m = 8$; $\bar{x} = 1{,}71$; $s_1 = 0{,}53$; $n = 10$; $\bar{y} = 2{,}53$; $s_2 = 0{,}87$; $\alpha = 0{,}01$

$H_0: \mu_1 - \mu_2 = 0 = \Delta_0$, Hypotes: $H_a: \mu_1 - \mu_2 < 0$ $(\mu_1 < \mu_2)$

$\Rightarrow$ Rejection region: $t \leq -t_{\alpha,\nu}$

$$t = \frac{\bar{x} - \bar{y} - \Delta_0}{\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}} \approx -2{,}46, \quad \nu = \frac{\frac{s_1^2}{m} + \frac{s_2^2}{n}}{\frac{(s_1^2/m)^2}{m-1} + \frac{(s_2^2/n)^2}{n-1}} = 15{,}1 \Rightarrow \text{Använd } \nu = 15$$

$\Rightarrow t_{\alpha,\nu} = 2{,}602 \Rightarrow t \not\leq -t_{\alpha,\nu} \Rightarrow$ Förkasta ej $H_0$! Facit säger annorlunda; det lär ha fel...

9.36 $H_0: \mu_D = \mu_1 - \mu_2 = 0 = \Delta_0$; $H_a: \mu_D > 0$; $\alpha = 0{,}01$; $n = 8$

$$\bar{d} = \bar{x} - \bar{y} = \frac{\sum x_i - \sum y_i}{n} = 7{,}25$$

$$s_D^2 = \frac{\sum (x_i - y_i)^2 - \frac{\left(\sum (x_i - y_i)\right)^2}{n}}{n-1} \Rightarrow s_D = 11{,}9$$

$$t = \frac{\bar{d} - \Delta_0}{s_D/\sqrt{n}} = 1{,}72; \quad t_{\alpha,\nu} = [\nu = n - 1 = 7] = 2{,}998$$

Rejection region: $t \geq t_{\alpha,\nu} \rightarrow$ Förkasta ej!

8.9 $H_0: p = 0{,}5$; $H_a: p \neq 0{,}5$; $n = 25$

a) $x = 6 \Rightarrow p = \frac{6}{25} = 0{,}24 \neq 0{,}5 \rightarrow$ Förkasta $H_0$!

8.25 ?

14.3 $n = 120$; $\alpha = 0{,}05$; $n_1 = 52$; $n_2 = 38$; $n_3 = 21$; $n_4 = 9$;

$p_{10} = 0{,}4$; $p_{20} = 0{,}3$; $p_{30} = 0{,}2$; $p_{40} = 0{,}1$; $k = 4$

$$\chi^2 = \sum_{i=1}^{4} \frac{(n_i - np_{i0})^2}{np_{i0}} = 1{,}569$$

Rejection region: $\chi^2 \geq \chi^2_{\alpha, k-1}$

$k-1 = 3 \Rightarrow \chi^2_{0,05;3} = 7{,}815 \Longrightarrow \chi \not\geq \chi_{\alpha, k-1} \Rightarrow$ Förkasta ej!

9.61 $n_1 = 23$; $s_1 = 32$; $n_2 = 20$; $s_2 = 54$; $\alpha = 0{,}05$

$H_0: \sigma_1^2 = \sigma_2^2$; $H_a: \sigma_1^2 \neq \sigma_2^2 \Longrightarrow$ Rejection region: $f \geq F_{\frac{\alpha}{2}, m-1, n-1}$
eller $f \leq F_{1-\frac{\alpha}{2}, m-1, n-1}$

$$f = \frac{s_1^2}{s_2^2} = 0{,}3512$$

$\frac{\alpha}{2} = 0{,}025$; $m-1 = 22$; $n-1 = 19$

I Appendix A9, använd $\nu_1 = 1$, $\nu_2 = 19$, $\alpha = 0{,}05 \Longrightarrow F_{0,025;22;19} \approx 2{,}16$

$$F_{1-\frac{\alpha}{2}, \nu_1, \nu_2} = 1 / F_{\frac{\alpha}{2}, \nu_1, \nu_2} \approx 0{,}463$$

$f \leq F_{1-\frac{\alpha}{2}, \nu_1, \nu_2} \Longrightarrow$ Förkasta $H_0$!

9.63 $m = 4$; $n = 4$; $1 - \alpha = 0{,}9 \Rightarrow \alpha = 0{,}1$

$$s_1^2 = \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{m}}{m-1} = 0{,}02576$$

$$s_2^2 = \frac{\sum y_i^2 - \frac{(\sum y_i)^2}{n}}{n-1} = 0{,}005492$$

X: Epoxy, Y: MMA..

$$P(F_{1-\frac{\alpha}{2}, \nu_1, \nu_2} < F < F_{\frac{\alpha}{2}, \nu_1, \nu_2}) = 0{,}9$$

$F_{0,05;3;3} = 9{,}28 = a$

$$F_{0,95;3;3} = 1 / F_{0,05;3;3} = 0{,}108 = b$$

$$F = \frac{s_2^2/\sigma_2^2}{s_1^2/\sigma_1^2} \Longrightarrow P\left(b < \frac{s_2^2\sigma_1^2}{s_1^2\sigma_2^2} < a\right) = P\left(\frac{s_1^2 b}{s_2^2} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{s_1^2 a}{s_2^2}\right) = 0{,}9$$

$$\Longrightarrow P\left(\frac{s_2^2}{s_1^2 a} < \frac{\sigma_2^2}{\sigma_1^2} < \frac{s_2^2}{s_1^2 b}\right) = 0{,}9 \Longrightarrow I_{\sigma_2^2/\sigma_1^2} = (0{,}023;\ 1{,}97)$$

9.55 $m=40,\ n=42;\ 1-\alpha=0{,}95 \Rightarrow \alpha=0{,}05$

Mönst 34: $x=22;\ y=29 \longrightarrow \hat{p}_1=0{,}55;\ \hat{p}_2=0{,}69$

$\hat{q}_1=0{,}45;\ \hat{q}_2=0{,}31$

$$I_{p_1-p_2}=\hat{p}_1-\hat{p}_2 \pm z\sqrt{\frac{\hat{p}_1\hat{q}_1}{m}+\frac{\hat{p}_2\hat{q}_2}{n}}$$

$$z_{\frac{\alpha}{2}}=1{,}96 \Rightarrow I_{p_1-p_2}=\underline{(-0{,}348;\ 0{,}0682)}$$

12.16 $n=15$ a) Ja

b) $$\hat{\beta}_1=\frac{\sum(x_i-\bar{x})(y_i-\bar{y})}{\sum(x_i-\bar{x})}=\underline{0{,}827} \quad (0{,}82697)$$

$$\hat{\beta}_2=\frac{\sum y_i-\hat{\beta}_1\sum x_i}{n}=\underline{-1{,}13} \quad (-1{,}12814)$$

c) $$y=\hat{\beta}_0+\hat{\beta}_1 x=[x=50]=\underline{40{,}22}$$

d) $$\hat{\sigma}^2=s^2=\frac{SSE}{n-2}=\frac{\sum(y_i-\hat{y}_i)^2}{n-2}=\frac{\sum(y_i-(\hat{\beta}_0+\beta_1 x_i))^2}{n-2}=27{,}46\ldots$$

$$\Rightarrow \hat{\sigma}=\underline{5{,}24}$$

e) $$r^2=1-\frac{SSE}{SST}=1-\frac{\sum(y_i-(\hat{\beta}_0+\hat{\beta}_1 x_i))^2}{\sum y_i^2-\frac{(\sum y_i)^2}{n}}=\underline{0{,}975}$$

12.32 $n=15;\ \alpha=0{,}05$?

$$H_0: \beta_1=0;\ H_a: \beta_1\neq 0 \Rightarrow \text{Rejection region: } \begin{cases} t\geq t_{\frac{\alpha}{2},n-2} \\ t\leq -t_{\frac{\alpha}{2},n-2} \end{cases}$$

$$t=\frac{\hat{\beta}_1}{s_{\hat{\beta}_1}}=\begin{bmatrix} s_{\hat{\beta}_1}=0{,}03652 \\ \beta_1=0{,}82697 \end{bmatrix}=22{,}64$$

$$t_{\frac{\alpha}{2},n-2}=2{,}16 \Rightarrow t>t_{\frac{\alpha}{2},n-2} \Rightarrow \text{Förkasta } H_0!$$

$$I_{\beta_1}=\hat{\beta}_1\pm t_{\frac{\alpha}{2},n-2}s_{\hat{\beta}_1}=\underline{(0{,}75;\ 0{,}91)}$$

12.39 $n=20$ Annova:

$$t^2=f=\frac{SSR}{SSE/(n-2)}=\frac{SST-SSE}{SSE/(n-2)}$$

$$SST=\sum y_i^2-\frac{(\sum y_i)^2}{n}=39{,}828;\quad SSE=\sum(y_i-(\hat{\beta}_0+\hat{\beta}_1 x_i))^2$$

$$\hat{\beta}_1=\frac{\sum(x_i-\bar{x})(y_i-\bar{y})}{\sum(x_i-\bar{x})^2}=0{,}041\ldots,\quad \beta_0=\frac{\sum y_i-\hat{\beta}_1\sum x_i}{n}=74{,}95\ldots \Rightarrow SSE=7{,}967$$

$$\Rightarrow t=\underline{8{,}48}$$

FORTSÄTTNING 12.39

t-test:

$$t = \frac{\hat{\beta}_1}{s_{\hat{\beta}1}}$$

$$s_{\hat{\beta}1} = \frac{s}{\sqrt{S_{xx}}}, \quad \hat{\sigma}^2 = s^2 = \frac{\sum y_i - \hat{\beta}_0 \sum y_i - \hat{\beta}_1 \sum x_i y_i}{n-2} \Rightarrow s = 0{,}6653\ldots$$

$$S_{xx} = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = 18\,921,\ldots$$

$$\Rightarrow s_{\hat{\beta}1} = \underline{0{,}004837}$$

$$\Rightarrow t = \underline{8{,}48} \Rightarrow \text{Check!}$$