Studienämnden Kf / Kb

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TRANSPORTPROCESSER

HÄRLEDNINGAR

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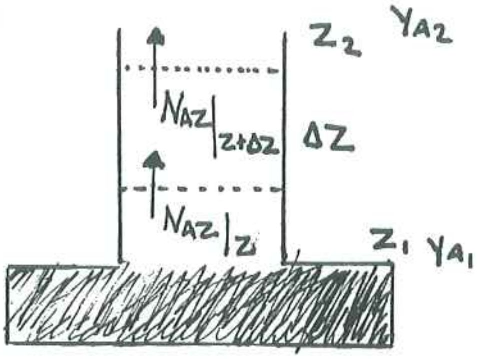
Arnoldcell



$$N_{Az} = - c D_{AB} \frac{dy_A}{dz} + y_A (N_{Az} + N_{Bz})$$

Stagnant gas: $N_{Bz} = 0$

$$\Rightarrow \quad N_{Az} = - \frac{c D_{AB}}{1 - y_A} \frac{dy_A}{dz}$$

$$\frac{dN_{Az}}{dz} = 0$$

$$\Rightarrow \quad \frac{d}{dz}\left(- \frac{c D_{AB}}{1 - y_A} \cdot \frac{dy_A}{dz} \right) = 0 \Rightarrow$$

$$\frac{c D_{AB}}{1 - y_A} \frac{dy_A}{dz} + C_1 = 0$$

$$C_1 dz = - \frac{1}{1 - y_A} dy_A \quad \Rightarrow \quad C_1 z = \ln(1 - y_A) + C_2$$

Gränser: $z_1 \longrightarrow z_2 \qquad y_{A1} \longrightarrow y_{A2}$

$$\Rightarrow \quad \frac{(1 - y_A)}{(1 - y_{A1})} = \left(\frac{1 - y_{AL}}{1 - y_{A1}} \right)^{\left(\frac{z - z_1}{z_2 - z_1} \right)}$$

Hastighetsprofil vid fullt utvecklad laminär strömning nedför ett lutande plan

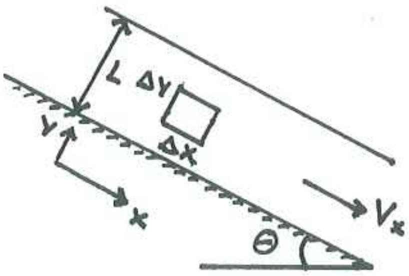
Rörelsemängdsbalans : $\sum F_x = \iint_{CS} V_x \rho \cancelto{(1)}{(V \cdot n)} dA + \frac{d}{dt} \iiint_{CV} \rho \cancelto{(2)}{V_x} dV$

(1) in = ut

(2) fullt utvecklad strömning

$$\sum F_x = \underbrace{P\Delta y\Big|_x \cancel{- P\Delta y\Big|_{x+\Delta x}}}_{(2)} + \tau_{yx}\Delta x\Big|_y - \tau_{yx}\Delta x\Big|_{y+\Delta y} + \rho g \Delta x \Delta y \sin\theta$$

Dividera med $\Delta x \Delta y \Rightarrow -\frac{d\tau_{yx}}{dy} - \rho g \sin\theta = 0$



Integration $\Rightarrow \tau_{yx} = C_1 - \rho g y \sin\theta$

$\tau_{yx}(L) = 0 \Rightarrow C_1 = \rho g L \sin\theta$

$\tau_{yx} = \rho g (L - y) \sin\theta = \mu \frac{dV_x}{dy}$

Integration $\Rightarrow V_x = \frac{\rho g}{\mu}(Ly - \frac{y^2}{2}) \sin\theta + C_2$

$V_x(0) = 0 \Rightarrow C_2 = 0$

$$V_x = \frac{\rho g}{\mu}(Ly - \frac{y^2}{2}) \sin\theta$$

Hastighetsprofil i rör med axel

rörelsemängdsbalans: $\sum F_x = \iint_{CS} \rho v_x (\cancelto{(1)}{v \cdot n})\, dA + \frac{d}{dt} \iiint_{CV} \rho \cancelto{(2)}{v_x}\, dV$

(1) in = ut

(2) fullt utvecklad strömning

Endast friktionsförluster ⇒

$$\sum F_x = \tau_{rx} (2\pi r \Delta x) \Big|_{r+\Delta r} - \tau_{rx} (2\pi r \Delta x) \Big|_{r} = 0$$

Dividera med $2\pi \Delta x \Delta r$

$$\Rightarrow \frac{d}{dr}(r \tau_{rx}) = 0 = \frac{d}{dr}\left(r \mu \frac{dv_x}{dr}\right)$$

Integration ⇒ $r\mu \frac{dv_x}{dr} + C_1 = 0 \Rightarrow dv_x = -\frac{C_1}{\mu} dr$

Integration ⇒ $v_x = -\frac{C_1}{\mu} \ln(r) + C_2$

$v_x(R_2) = 0 \Rightarrow C_2 = -\frac{C_1}{\mu} \ln(R_2)$

$v_x(R_1) = v \Rightarrow v = \frac{C_1}{\mu} \ln(R_1) - \frac{C_1}{\mu} \ln(R_2) = \frac{C_1}{\mu}\left(\ln \frac{R_1}{R_2}\right)$

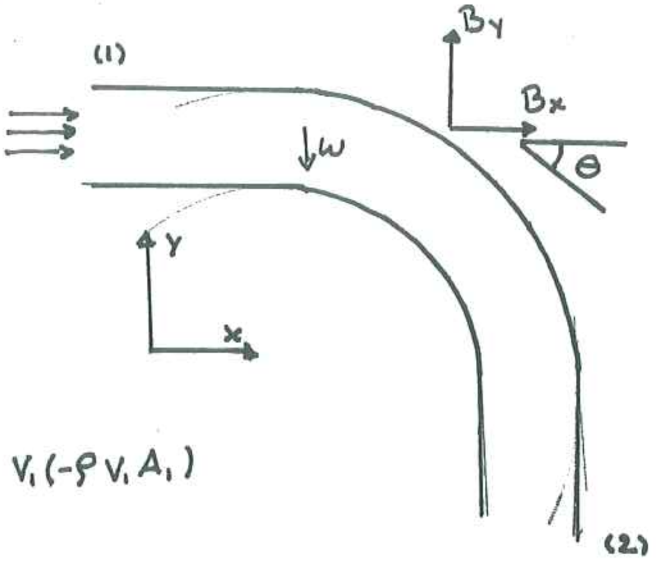
$$\Rightarrow \frac{C_1}{\mu} = \frac{v}{\ln \frac{R_1}{R_2}}$$

$$\Rightarrow v_x(r) = v \frac{\ln(r/R_2)}{\ln(R_1/R_2)}$$

kraftkomposanter i en rörböj

$$\sum F_x = P_1 A_1 - P_2 A_2 \cos\theta + B_x$$

$$\sum F_y = P_2 A_2 \sin\theta - \omega + B_y$$



$$\iint_{Cs} v_x \rho (\mathbf{V} \cdot \mathbf{n}) \, dA = (v_2 \cos\theta)(\rho v_2 A_2) + v_1(-\rho v_1 A_1)$$

$$\iint_{CS} v_y \rho (\mathbf{V} \cdot \mathbf{n}) \, dA = (-v_2 \sin\theta)(\rho v_2 A_2)$$

fullt utvecklad strömning $\Rightarrow \frac{d}{dt} \iiint_{CV} = 0$

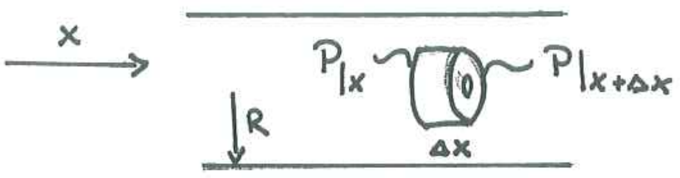
$$P_1 A_1 - P_2 A_2 \cos\theta + B_x = (v_2 \cos\theta)(\rho v_2 A_2) + v_1(-\rho v_1 A_1)$$

$$P_2 A_2 \sin\theta - \omega + B_y = (-v_2 \sin\theta)(\rho v_2 A_2)$$

$$B_x = v_2^2 \rho A_2 \cos\theta - v_1^2 \rho A_1 - P_1 A_1 + P_2 A_2 \cos\theta$$

$$B_y = -v_2^2 \rho A_2 \sin\theta - P_2 A_2 \sin\theta + \omega$$

Hastighetsprofil vid fullt utvecklad, horisontell, laminär rörströmning



Rörelsemängdsbalans: $\sum F_x = \iint_{CS} \rho v_x \overset{(1)}{\cancel{(v \cdot n)}}\, dA + \overset{(2)}{\cancel{\frac{d}{dt}}} \iiint_{CV} \rho v_x \, dV$

(1) in = ut

(2) fullt utvecklad strömning

$$\sum F_x = P 2\pi r \Delta r \Big|_x - P 2\pi r \Delta r \Big|_{x+\Delta x} + \tau_{rx} 2\pi r \Delta x \Big|_{r+\Delta r} - \tau_{rx} 2\pi r \Delta x \Big|_r$$

dividera med $2\pi \Delta r \Delta x$

$$\Rightarrow \quad -r \frac{dP}{dx} + \frac{d}{dr}(r \tau_{rx}) = 0$$

integration $\Rightarrow \quad \tau_{rx} r = \frac{dP}{dx} \frac{r^2}{2} + C_1 \quad \Rightarrow \quad \tau_{rx} = \frac{dP}{dx} \frac{r}{2} + \frac{C_1}{r}$

C_1 måste vara noll

för att undvika att $\frac{C_1}{r} \rightarrow \infty$

$$\tau_{rx} = \mu \frac{dv_x}{dr} \quad \Rightarrow \quad \mu \frac{dv_x}{dr} = \frac{dP}{dx} \frac{r}{2}$$

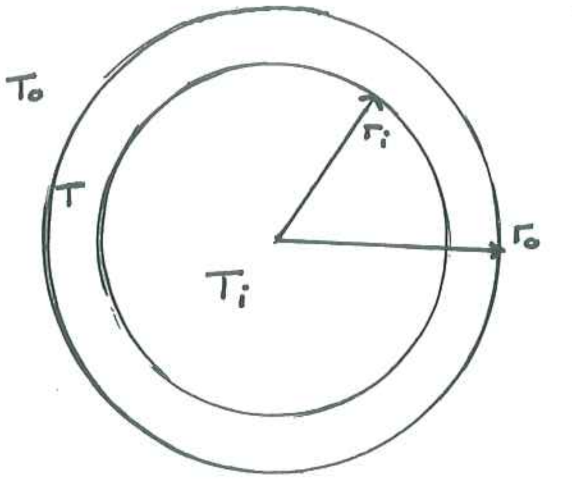
integration $\Rightarrow \quad v_x = \frac{dP}{dx} \frac{1}{2\mu} \frac{r^2}{2} + C_2$

$v_x(R) = 0 \quad \Rightarrow \quad C_2 = \frac{1}{4\mu} \frac{dP}{dx} R^2$

$$\Rightarrow \quad v_x = \frac{dP}{dx} \frac{1}{4\mu} (r^2 - R^2)$$

Medelarea för stationär vårmeledning genom cylindriskt skal

$$q = \frac{k\bar{A}}{r_o - r_i}\Delta T$$



$$q = -kA\frac{dT}{dr} \qquad A = 2\pi r L$$

integrering $r_i \to r_o$

$T_i \to T_o$ $\qquad q$ = konstant

$$\ln\left(\frac{r_o}{r_i}\right) = -\frac{2\pi L k}{q}(T_o - T_i)$$

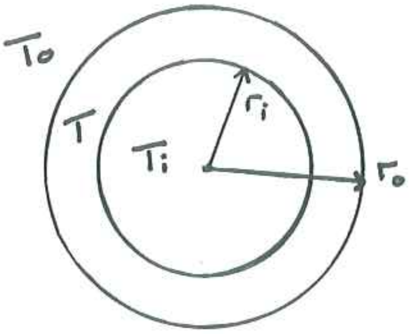
$$\Rightarrow \quad q = \frac{2\pi L k(T_i - T_o)}{\ln(r_o/r_i)} = \frac{2\pi L k(T_i - T_o)(r_o - r_i)}{\ln(r_o/r_i)\;(r_o - r_i)} = \frac{k\bar{A}}{r_o - r_i}\Delta T$$

$$\Rightarrow \quad \bar{A} = \frac{2\pi L(r_o - r_i)}{\ln(r_o/r_i)}$$

Temperaturprofil i cylindriskt skal

$q = kA\frac{dT}{dr}$ $\quad$ $A = 2\pi r L$

$\Rightarrow \frac{q}{L} = \frac{2\pi k}{\ln(r_o/r_i)} (T_i - T_o)$



$\cancelto{0}{\text{Ackumulation}} = \text{in} - \text{ut} + \cancelto{0}{\text{producerat}}$

$\Rightarrow \frac{dq}{dr} = 0$

$\Rightarrow \frac{d}{dr}\left(\frac{r\,dT}{dr}\right) = 0$ $\quad$ enligt definition

integration $\Rightarrow r\frac{dT}{dr} = -C_1 \Rightarrow T = -C_1 \ln(r) + C_2$

$T(r_i) = T_i \Rightarrow C_2 = T_i + C_1 \ln(r_i)$

$T(r_o) = T_o \Rightarrow C_1 = \frac{T_o - T_i}{\ln(r_i/r_o)}$

$\Rightarrow T = \frac{T_o - T_i}{\ln(r_o/r_i)} \ln(r) + T_i + \frac{T_o - T_i}{\ln(r_o/r_i)} \ln(r_i)$

$\Rightarrow T = T_i + \frac{T_o - T_i}{\ln(r_o/r_i)} \ln(r/r_i)$