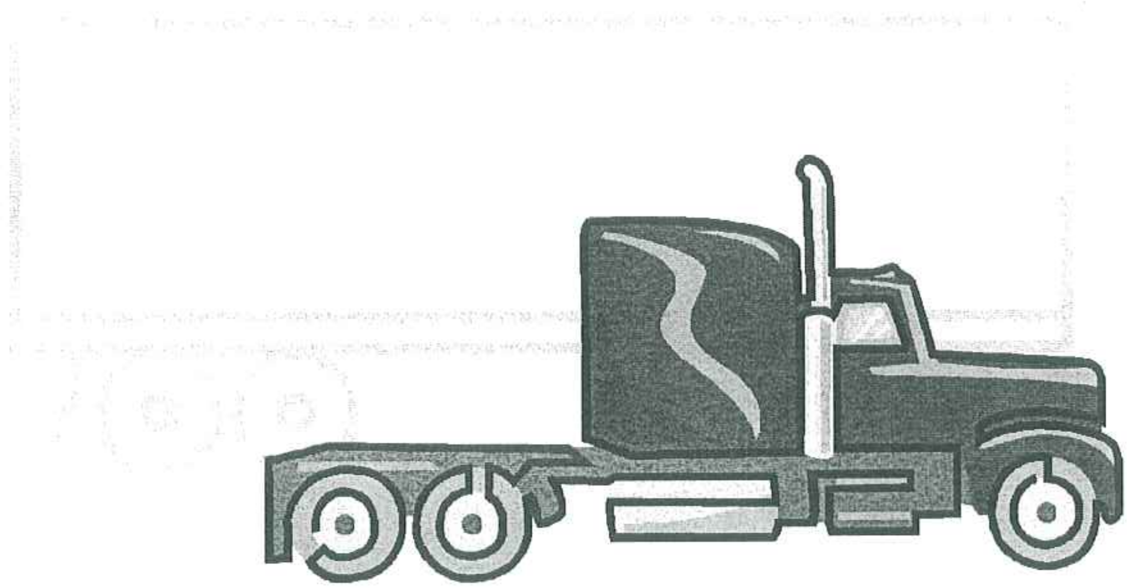
Studienämnden Kf / Kb

TRANSPORTPROCESSER

LÖSNINGAR



Av: Putte kf-06

December 2008

1.1} $P = \rho v^2 \left(\sin(x/a) \sin(y/b) + \frac{2x}{a} \right)$ "tryckfält"

ρ, v, a, b är konstanter

Bestäm $\nabla P(ab)$

$$P'_x = \rho v^2 \left(\frac{1}{a} \cos(\overbrace{x/a}^{\alpha}) \sin(\overbrace{y/b}^{\beta}) + \frac{2}{a} \right)$$

$$= \rho v^2 \left(\frac{1}{2a} \left[\sin(\alpha - \beta) + \sin(\alpha + \beta) \right] + \frac{2}{a} \right)$$

$$= \rho v^2 \left(\frac{1}{2a} \left[\sin\left(x/a - y/b\right) + \sin\left(x/a + y/b\right) \right] + \frac{2}{a} \right)$$

$$= \rho v^2 \left(\frac{1}{2a} \sin(2) + \frac{2}{a} \right)$$

$$= \rho v^2 \left[\frac{1}{a} \left(\frac{\sin(2)}{2} + 2 \right) \right]$$

$$P'_y = \rho v^2 \left(\sin(x/a) \cos(y/b) \right)$$

$$= \rho v^2 \left(\frac{1}{b} \cdot \frac{\sin(2)}{2} \right)$$

$$\Rightarrow \quad \underline{\underline{\nabla P = \rho v^2 \left[\frac{1}{a} \left(\frac{\sin(2)}{2} + 2 \right) \vec{e}_x + \frac{1}{b} \left(\frac{\sin(2)}{2} \right) \vec{e}_y \right]}}$$

1.2)

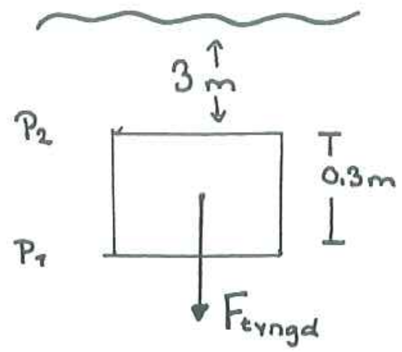
FLÖDE

- skjuvspänning
- tryckgradient
- hastighet

FLUID

- ρ
- c_p
- P
- T
- ljudhastighet

1.3}



$$\Delta P = -\rho g (h_2 - h_1)$$

$$\sum F_y = 0$$

$$\Rightarrow -F_{tyngd} - A\Delta P + F_o = 0$$

$$\Rightarrow F_o = \rho_K g V - A(\rho_f g (h_2 - h_1))$$

a)

$$\left.\begin{aligned} \rho_f &= 1000 \ kg/m^3 \\ \rho_K &= 160 \ kg/m^3 \\ g &= 9.81 \ m/s^2 \\ V &= 0.3^3 \ m^3 \end{aligned}\right\} \quad F_o = \underline{\underline{-222 \ N}}$$

b)

$$\left.\begin{aligned} \rho_f &= 1000 \ kg/m^3 \\ \rho_K &= 7800 \ kg/m^3 \\ g &= 9.81 \ m/s^2 \\ V &= 0.3^3 \ m^3 \end{aligned}\right\} \quad F_o = \underline{\underline{1801 \ N}}$$

1.4) $\Delta P = \rho g h$

$\Delta P = 1 \text{ bar} = 10^5 \text{ Pa}$

$g = 9.82 \text{ m/s}^2$

$\rho = m/V$

$m = n \cdot M$

$n = \frac{PV}{RT}$

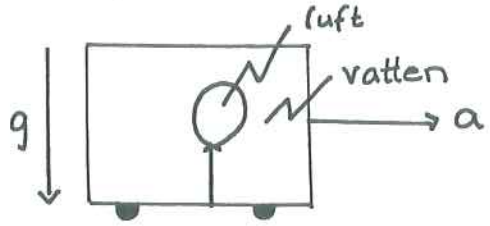
$M = 30$ antar 50% syre = 50% kväve

$\Rightarrow \rho = 1.21 \text{ kg/m}^3$

$h = \frac{\Delta P}{\rho g}$

$= \underline{\underline{8461 \text{ m}}}$

1.5}



1, $\sum F_y = 0$

$\Rightarrow F_{lyft} - mg - F_{snöre} = 0$

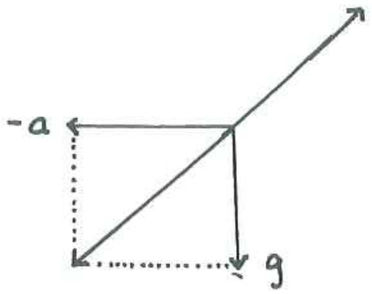
$F_{snöre} = mg - F_{lyft}$

$= \rho_l V_b g - \rho_{H2O} g V_b$

$= g V_b (\rho_l - \rho_{H2O})$

2, $\sum F = ma$

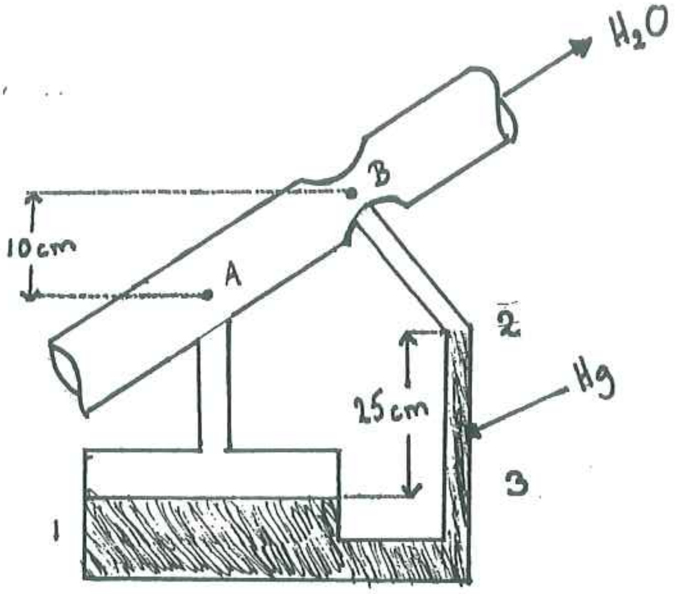
$\Rightarrow \nabla P = \rho(g-a)$ (se 2-3 : www)



lyftkraften måste motverka resulterande tyngdkraften

$\Rightarrow$ ballongen rör sig framåt

1.6}



$\rho_{Hg} = 13\,500 \text{ kg/m}^3$

$$\nabla P = g\rho \Rightarrow \frac{dP}{dx} = -g_x \rho \Rightarrow \Delta P = -g \rho \Delta x$$

$$P_1 - P_A = -g \rho_{H2O} (x_1 - x_A) \qquad (1)$$

$$P_3 - P_2 = -g \rho_{Hg} (x_3 - x_2) \qquad (2)$$

$$P_2 - P_B = -g \rho_{H2O} (x_2 - x_B) \qquad (3)$$

$$P_3 = P_1 \qquad (4)$$

$$\left.\begin{array}{l} P_3 = P_1 = \rho_{Hg}\, g\, (x_2 - x_3) + P_2 \\ P_2 = \rho_{H2O}\, g\, (x_B - x_2) + P_B \end{array}\right\} \text{Sätts in i (1)}$$

$$\Rightarrow P_A - P_B = \rho_{Hg}\, g\, (x_2 - x_3) + \rho_{H2O}\, g \left((x_1 - x_2) + (x_B - x_A) \right)$$

$$= 31670 \text{ Pa} \approx \underline{\underline{31.7 \text{ kPa}}}$$

1.8} $v_1 A_1 = v_2 A_2$

$d_1 = 0.1$ m

$v_1 = 0.6$ m/s

$d_2 = 0.01$ m

$$v_2 = \frac{v_1 A_1}{A_2} = \frac{v_1 (d_1/2)^2}{(d_2/2)^2} = \underline{\underline{60}} \text{ m/s}$$

$$a_1 A_1 = a_2 A_2$$

$a_1 = 1.5$ m/s^2

$$a_2 = \frac{a_1 (d_1/2)^2}{(d_2/2)^2} = \underline{\underline{150}} \text{ m/s}^2$$

2.1 } $V_1 = 91 \text{ m/s}$ $\quad V_2 = 274 \text{ m/s}$ $\quad \rho = 1.29 \text{ kg/m}^3$

$A_1 = 1 \text{m}^2$ $\quad A_2 = 1 \text{ m}^2$

Impulsekvationen

$$\sum F = \iint \mathbb{V} \cdot \rho (\mathbb{V} \cdot n)\, dA + \frac{d}{dt} \iiint \rho \mathbb{V}\, dV \qquad (5\text{-}4)$$

Stationärt tillstånd i x-led $\Rightarrow$ [bränsletillförsel : $\rho_1 \neq \rho_2$]

$$F_x = \iint_{A_1} V_1 \rho_1 (V_1 (-1))\, dA_1 + \iint_{A_2} V_2 \rho_2 (V \cdot (1))\, dA_2 \quad \Rightarrow$$

$$F_x = -V_1^2 \rho_1 A_1 + V_2^2 \rho_2 A_2 \qquad (1)$$

sök ρ_2 :

$$\dot{m}_{ut} = 1.01\, \dot{m}_{in} \qquad \dot{m} = V \cdot \rho \cdot A$$

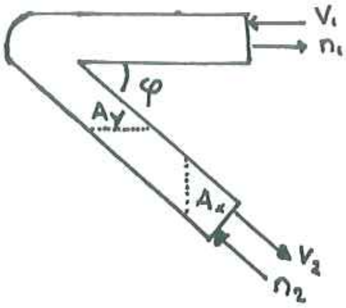
$$\dot{m}_{ut} = \rho_2 V_2 A_2 = 1.01 \cdot \rho_1 V_1 A_1$$

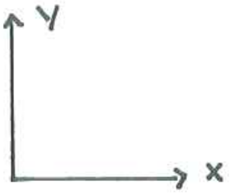
$$\Rightarrow \rho_2 = \frac{1.01 \cdot V_1 \rho_1 A_1}{V_2 A_2} \qquad (2)$$

(2) i (1) ger: $F_x = V_2 \cdot V_1 \cdot 1.01 \cdot \rho_1 \cdot A_1 - V_1^2 \rho_1 A_1$

$$= 21\,800 \text{ N} = \underline{\underline{21.8 \text{ kN}}}$$

2.2





$q = 0.057 \ m^3/s$

$V = 7.6 \ m/s$

$\Rightarrow A = 0.0075$

$$\cos(30) = \frac{A}{A_x} \Rightarrow A_x = \frac{A}{\cos(30)}$$

$$\sin(30) = \frac{A}{A_y} \Rightarrow A_y = \frac{A}{\sin(30)}$$

0: stationärt

$$\cancel{\frac{\partial}{\partial t}} \iiint_{C.V} \rho \vec{v} \, dV = - \iint_{C.S} \vec{v}(\rho \vec{v} \cdot n) \, dA + \sum \vec{F}$$

$$\sum \vec{F} = \iint_{C.S} \vec{v} \rho (\vec{v} \cdot n) \, dA$$

$$= \iint_{A_1} \vec{v}_1 \rho (\vec{v}_1 \cdot \vec{n}_1) \, dA + \iint_{A_2} \vec{v}_2 \rho (\vec{v}_2 \cdot \vec{n}_2) \, dA$$

$$= \rho v_x^2 \iint_{A_1} dA + \rho v_y^2 \iint_{A_2} dA$$

$$\sum F_x = \rho V_x^2 A_1 + \rho (V_2 \cdot \cos(30))^2 \cdot \frac{A_1}{\cos(30)}$$

$$= \rho V_x^2 A_1 + \rho V_2^2 \cos(30) A_1$$

$= 808\ N$ kraften på fluiden

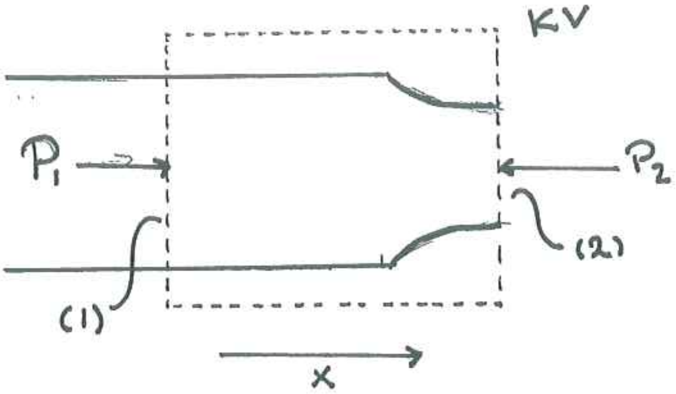
$\Rightarrow \underline{\underline{-808 N}}$ kraften på rörböjen har motsatt tecken

$$\sum F_y = \rho (V_y \cdot \sin(30))^2 \frac{A}{\sin(30)}$$

$$= -\rho V_y^2 \cdot \sin(30) \cdot A$$

$$= \underline{\underline{-216.6 N \approx 217 N}}$$

2.3



verkar på hela arean alltså A_1

kontrollvolymen läggs utanför munstycket

$$\sum F_x = \iint V_x \rho (V \cdot n)\, dA = [2.1] = \rho V_2^2 A_2 - \rho V_1^2 A_1$$

$$\sum F_x = F_{tryck} + F_{insp} = P_1 A_1 - P_2 A_2 + F_{insp}$$

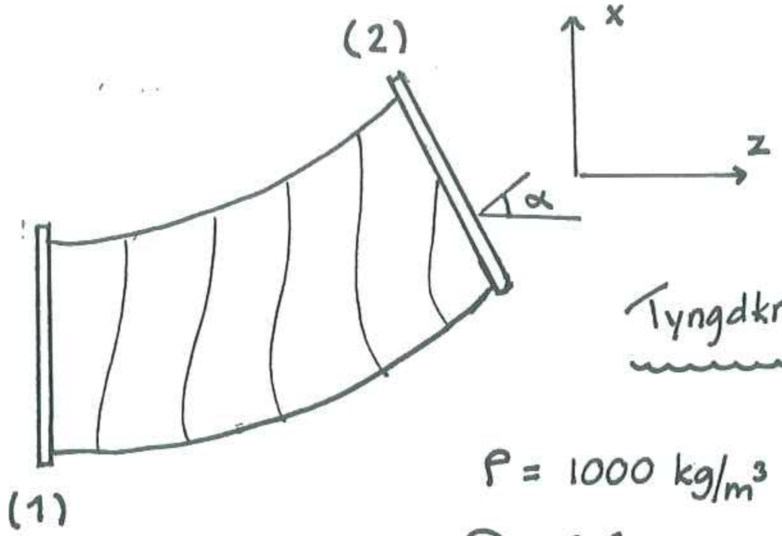
F_{insp} är den sökta kraften som krävs för att hålla fast munstycket

$$\Rightarrow (P_1 - P_2) A_1 + F_{insp} = \rho V_2^2 A_2 - \rho V_1^2 A_1$$

$$Q = V \cdot A \Rightarrow F_{insp} = \frac{4 \rho Q^2}{\pi} \left(\frac{1}{D_2^2} - \frac{1}{D_1^2} \right) - (P_1 - P_2) \frac{\pi}{4} D_1^2$$

$$= -1458\, N$$

2.4}



$\alpha = 30°$

Tyngdkraften försummas

$\rho = 1000\ kg/m^3$

$D_1 = 0.3\ m$ $\quad$ $D_2 = 0.38\ m$

$V_1 = 12\ m/s$ $\quad$ $P_0 = 101325\ Pa$

$P_1 = 128\ kPaö$ $\quad$ $P_2 = 145\ kPaö$

$A_1 = 0.07\ m^2$ $\quad$ $A_2 = 0.11\ m^2$

KE: $V_2 = V_1 \cdot \frac{A_1}{A_2} \approx 7.64\ m/s$

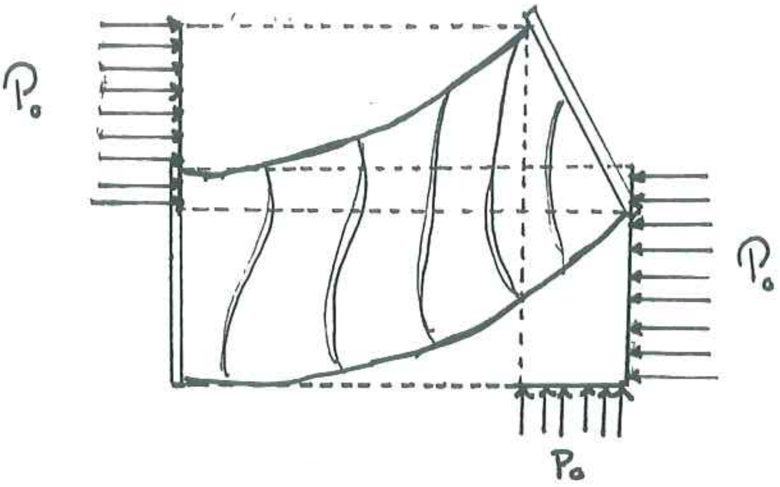
$$\Sigma F_x = \rho V_2^2 \cdot \sin\alpha \cdot A_2 \qquad = 3172.05\ N$$

$$\Sigma F_z = -\rho V_1^2 A_1 + \rho V_2^2 \cos\alpha\, A_2 \qquad = -4684.61\ N$$

$$F_z = \Sigma F_z + P_2 A_2 \cos\alpha - P_1 A_1$$

$$F_x = \Sigma F_x + P_2 A_2 \sin\alpha$$

} udda skrivsett, men finns värre saker att oroa sig för



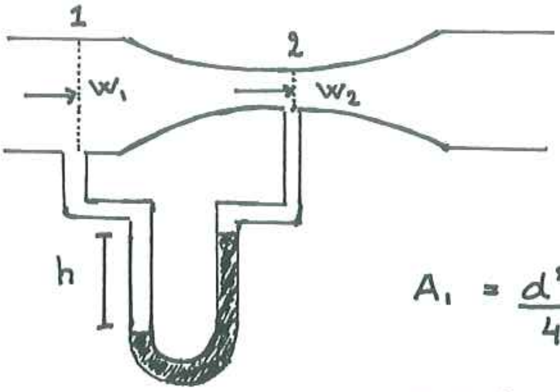
Balans på en kontrollvolym utanför systemet

$$\tilde{F}_z = F_z - P_0 A_2 \cos\alpha + P_0 A_1$$

$$= \underline{\underline{509.1\ N}}$$

$$\tilde{F}_x = F_x - P_0 A_2 \sin\alpha = \underline{\underline{11394\ N}}$$

3.1



$d = 0.6\ m$

$q = 6\ m^3/s$

$h = 0.1\ m$

$\rho_l = 1.23\ kg/m^3$

$\rho_a = 800\ kg/m^3$

$A_1 = \frac{d^2\pi}{4}$

$v = q/A_1 = 21.22\ m/s$

$\Delta P = \rho g h = 784.8\ Pa$

$$\begin{cases} BE: P_1 + \frac{\rho v_1^2}{2} = P_2 + \frac{\rho v_2^2}{2} \\ KE: v_1 A_1 = v_2 A_2 \end{cases}$$

$$\Rightarrow \quad P_1 + \frac{\rho v_1^2}{2} = P_2 + \frac{\rho \left(\frac{v_1 A_1}{A_2}\right)^2}{2}$$

$$2\Delta P + \rho v_1^2 = \rho \left(\frac{v_1 A_1}{A_2}\right)^2$$

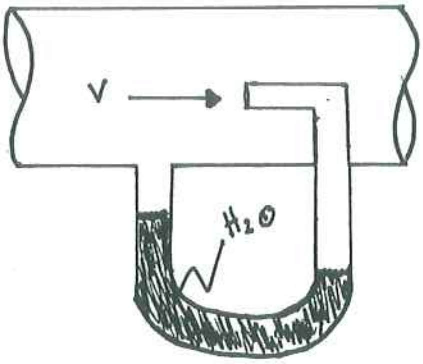
$$\Rightarrow \quad \frac{v_1 A_1}{A_2} = \sqrt{\frac{2\Delta P}{\rho} + v_1^2} \quad \Rightarrow \quad A_2 = \frac{v_1 A_1}{\sqrt{\frac{2\Delta P}{\rho} + v_1^2}}$$

$$= \frac{d^2\pi}{4}$$

$$\Rightarrow \quad d = \sqrt{\frac{4A_2}{\pi}} = \underline{\underline{0.43 m}}$$

3.2}

a/



$v = 15$ m/s

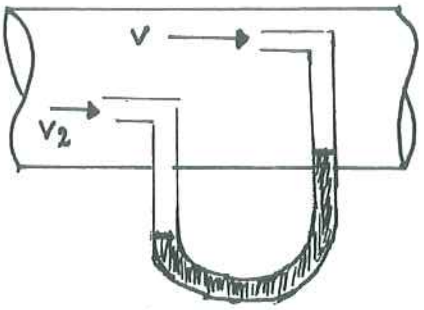
$\rho = S.G. \cdot \rho_{H_2O}$

$$P_0 = P_1 + \frac{\rho v_1^2}{2}$$

$$\frac{\rho v^2}{2} = \Delta P = 136.125$$

$$\Delta P = \rho g h \quad \Rightarrow \quad h = \underline{\underline{1.4 \text{ cm}}}$$

b/



$v_1 = 15 \text{ m/s}$

$v_2 = 24 \text{ m/s}$

$\rho_0 = 860 \text{ kg/m}^3$

$$\Delta P_1 = \frac{\rho v_1^2}{2} \qquad \Delta P_2 = \frac{\rho v_2^2}{2} \qquad \Delta P = \rho_0 g h$$

$$h = \frac{\Delta P}{\rho_0 g} = \frac{\Delta P_2 - \Delta P_1}{\rho_0 g} = \frac{\rho (v_2^2 - v_1^2)}{2 \rho_0 g}$$

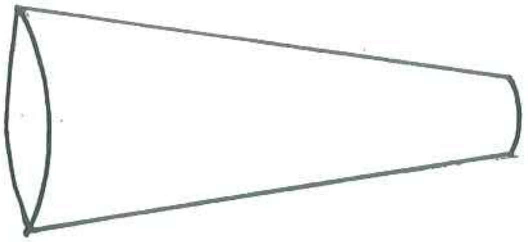
$\Rightarrow \quad h = \underline{\underline{2.5 \text{ cm}}}$

3.3

$d_1 = 0.1\,m$

$v_1 = 2\,m/s$

$P_1 = 0,32\,bar$



$d_2 = ?$

$P_{vap} = 0.02\ bar$

$\rho = 1000\ kg/m^3$

$$P_1 + \frac{\rho v_1^2}{2} = P_{vap} + \frac{\rho v_2^2}{2} \quad \Rightarrow \quad v_2^2 = \frac{2\Delta P}{\rho} + v_1^2$$

$$v_1 A_1 = v_2 A_2$$

$$\Rightarrow \quad \frac{v_1 A_1}{A_2} = \sqrt{\frac{2\Delta P}{\rho} + v_1^2}$$

$$\Rightarrow \quad A_2 = \frac{v_1 A_1}{\sqrt{\frac{2\Delta P}{\rho} + v_1^2}}$$

$$= 0.00785$$

$$= \frac{d_2^2 \pi}{4}$$

$$\Rightarrow \quad d_2 = \underline{\underline{5\ cm}}$$

3.4}

$\rho_{Hg} = 13500 \ kg/m^3$

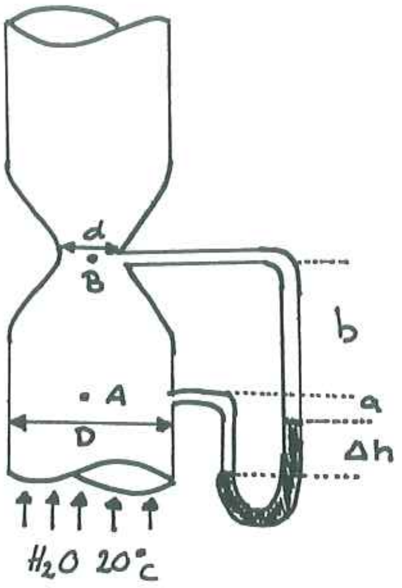
$\Delta h = 0.15 \ m$

$d = 0.15 \ m$

$a = 0.4 \ m$

$b = 0.2 \ m$

$D = 0.3 \ m$



$$\Delta P = \rho g \Delta h$$

$$V_A A_A = V_B A_B \quad \Rightarrow \quad \frac{V_A}{V_B} = 0.25 \quad \Rightarrow \quad V_B = 4 V_A$$

$$P_A - P_B + \frac{\rho}{2}(V_A^2 - V_B^2) = \rho g (h_B - h_A)$$

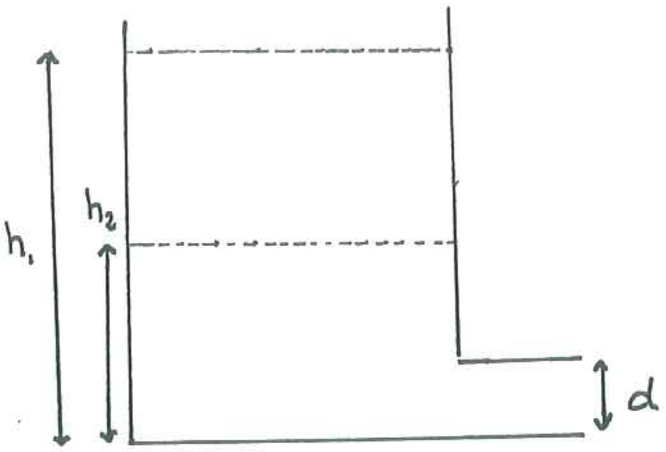
$$\frac{2\Delta P}{\rho} + V_A^2 - 16 V_A^2 = 2gb$$

$$15 V_A^2 = \frac{2\Delta P}{\rho} - 2gb$$

$$V_A = \sqrt{\frac{2\Delta P}{15\rho} - \frac{2gb}{15}}$$

$$Q = V_A \cdot A_A \quad \Rightarrow \quad \underline{\underline{Q = 0.11 \ m^3/s}}$$

3.15}



D = 3 m

d = 0,05 m

h_1 = 8.5 m

h_2 = 1.2 m

$$\iint \rho(\mathbb{V}\cdot\mathbb{n})\,dA + \frac{d}{dt}\iiint \rho\,dV = 0$$ inkompressibelt ⇒

$$\iint (\mathbb{V}\cdot\mathbb{n})\,dA + \frac{d}{dt}\iiint dV = 0$$

$$\left.\begin{aligned} & V_2 A_2 + V_1(-1)A_1 + \frac{d}{dt}V = 0 \\ & V = \frac{\pi D^2}{4} h \end{aligned}\right\} \quad V_2 A_2 - V_1 A_1 + \frac{\pi D^2}{4}\cdot\frac{dh}{dt} = 0 \quad (1)$$

$V_1 = 0$: V_2 fås ur Bernoulli

$$g y_1 \rho = g y_2 \rho + \frac{V_2^2 \rho}{2} \quad \Rightarrow$$

$$V_2 = \sqrt{2g\Delta y}$$

$$\Rightarrow \quad \frac{\pi d^2}{4}\sqrt{2g\Delta h} + \frac{\pi D^2}{4}\frac{dh}{dt} = 0 \qquad \Rightarrow$$

$$\frac{dh}{dt} + \left(\frac{d}{D}\right)^2 \sqrt{2g}\cdot\sqrt{h}$$

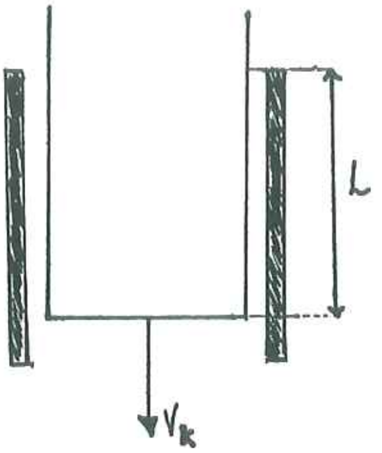
: 1:ordningens separabel differential ekvation

$$\frac{dh}{dt} = -c\sqrt{h} \qquad \Rightarrow$$

$$-ct = \left[2\sqrt{h}\right]_{h_1}^{h_2} \quad \Rightarrow \quad t = \frac{1}{-c}\,2\left(\sqrt{h_2} - \sqrt{h_1}\right)$$

$$\Rightarrow \quad t = 2956 = \underline{\underline{49{,}3 \text{ min}}}$$

4.1



$d = 35.56$ cm

$D = 35.58$ cm

$\nu = 3.7 E^{-4}$ m^2/s

$\rho = 850$ kg/m^3

$V_k = 0.15$ m/s

$L = 2.44$ m

$$F_{skjuv} = \tau \cdot A \Rightarrow \left(A\mu \cdot \frac{dV}{dx} \right)_{kolv} = \left(A\mu \frac{dV}{dx} \right)_{cyl} \Rightarrow$$

$$A_k \left(\frac{dV}{dx} \right)_k > A_c \left(\frac{dV}{dx} \right)_c \qquad \text{ty } A_k < A_c$$

Antag $A_k \approx A_c \approx A_{medel} \Rightarrow \left(\frac{dV}{dx} \right)_k = \left(\frac{dV}{dx} \right)_c$

Linjär hastighetsprofil $\Rightarrow \left(\frac{dV}{dx} \right)_k = \frac{\Delta V}{\Delta X} = \frac{V_k - 0}{\frac{D-d}{2}}$

Friktionskraften på kolven är oljans skjuvkraft vid väggen

$$F_f = 2\pi d \cdot L \cdot \frac{2V_k}{D-d} \cdot \rho \cdot \nu = 1286 \text{ N}$$

4.2

$L = 2.44\ m$

$m = 680\ kg$

Kraftbalans på kolven: (ingen acceleration)

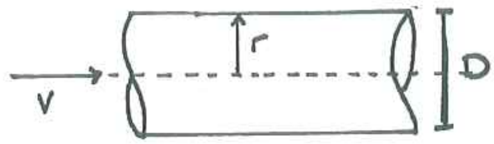
$-F + F_f = 0$ $\qquad F_f = \tau \cdot A$ enligt 4.1

$\Rightarrow -mg + \tau A = 0$

$\tau = \mu \frac{dv}{dx} = \mu \frac{2V_k}{D-d}$ enligt 4.1

$$\Rightarrow V_k = \frac{mg(D-d)}{2\mu d \pi L} = 0{,}779\ m/s$$

4.3



D = 2mm

$V_m = 0.6\,m/s$

$$V = V_{max}\left(1 - \frac{r^2}{R^2}\right)$$

$$\tau_{vägg} = \mu \frac{dV}{dr}\Big|_{r=R} = \mu \frac{d}{dr}\left(V_{max}\left(1 - \frac{r^2}{R^2}\right)\right)\Big|_{r=R} =$$

$$= -\mu V_{max} \frac{2r}{R^2}\Big|_{r=R} = -\mu V_{max} \frac{2}{R}$$

$$V_m = \frac{\int V_x \, dA}{A} = \frac{\int_0^{2\pi}\int_0^{R} V_{max}\left(1 - \frac{r^2}{R^2}\right) r\,dr\,d\theta}{\pi R^2}$$

$$= \frac{2V_{max}}{R^2}\int^{R}\left(r - \frac{r^3}{R^2}\right) dr = \frac{2V_{max}}{R^2}\left(\frac{R^2}{2} - \frac{R^2}{4}\right)$$

$$= \frac{V_{max}}{2} \Rightarrow V_{max} = 2V_m = 1.2\ m/s$$

$$\tau_{vägg} = -2\mu \frac{V_{max}}{R} = \underline{\underline{-2400\,\mu\ N/m^2}}$$

4.4 Hagen-Poiseuilles ger:

$$-\frac{dP}{dx} = 32\mu \frac{V_m}{D^2} \qquad Q = V_m \cdot \frac{\pi D^2}{4} \qquad \Rightarrow$$

$$-\frac{dP}{dx} = \frac{128\mu Q}{\pi D^4}$$

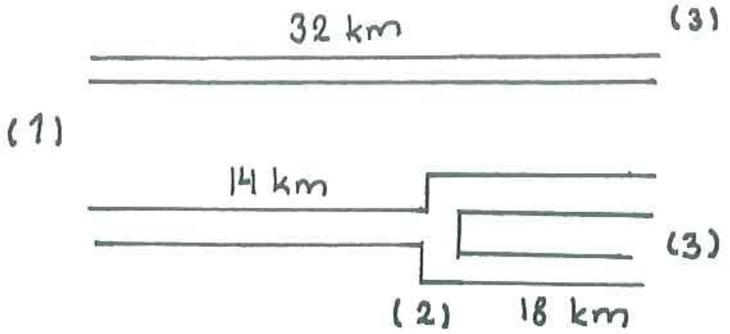
Fördubblad diameter:

$$Q = -\frac{dP}{dx}\frac{\pi D^4}{128\mu}$$

$Q_{ny} = 2^4 Q = 16\,Q$ dvs 16 ggr ökning

Flödet blir 16 ggr större om diametern dubblas

4.5



$Q = 2000\ m^3/dygn$

$\Delta P = 3.45\ MPa$

$L_{1-3} = 32\ km$

Antag laminärt flöde $\Rightarrow$

Hagen - Poiseulle: $-\frac{dP}{dx} = 32\mu\frac{V_m}{D^2}$

Integration ger: $\Delta P = 32\mu\frac{V_m \cdot L}{D^2} = \frac{128\mu L Q}{\pi D^4}$

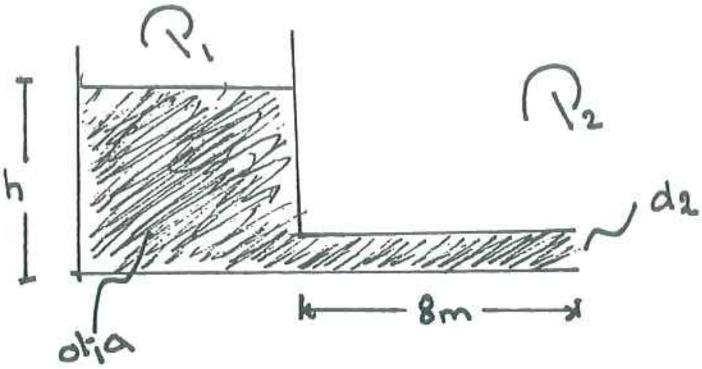
$$\Delta P_{1-3} = \frac{128\mu L_{1-3} Q}{\pi D^4}$$

$$\Delta P_{1-2-3} = \frac{128\mu L_{1-2} Q_{ny}}{\pi D^4} + \frac{128\mu L_{2-3} Q_{ny}}{2\pi D^4}$$

$$\Delta P_{1-3} = \Delta P_{1-2-3} \Rightarrow Q \cdot L_{1-3} = Q_{ny}\left(L_{1-2} + \frac{L_{2-3}}{2}\right)$$

$$Q_{ny} = \frac{Q \cdot L_{1-3}}{L_{1-2} + \frac{L_{2-3}}{2}} = \underline{\underline{2783\ m^3/dygn}}$$

4.6}



$\Delta P = 207$ kPa

$d_2 = 0.64 E^{-2}$ m

$\rho = 850$ kg/m^3

$\mu = 0{,}017$ Pas

oviskös ⇒ Bernoullis ekvation:

$$\rho g h + \Delta P = \rho \frac{v^2}{2} \quad \Rightarrow \quad v = 22.07 \text{ m/s}$$

$$Q = A \cdot v = \underline{\underline{0.71 E^{-3} \text{ m}^3/\text{s}}}$$

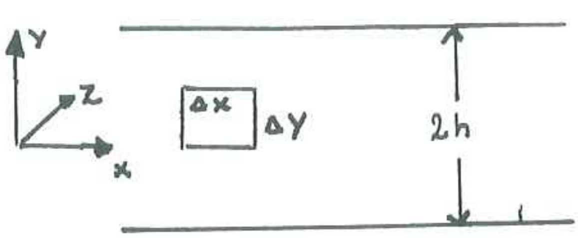
laminärt flöde ⇒ Hagen - Poiseulle :

$$Q = \frac{\pi D^4}{128 \mu} - \frac{\Delta P}{L} = \underline{\underline{6.27 E^{-5} \text{ m}^3/\text{s}}}$$

oviskös : $7.1 E^{-4}$ m^3/s

viskös : $0.627 E^{-4}$ m/s

4.7



Antag stationärt, inkompressibelt, fullt utvecklat flöde

Impulsekvationen på fluidelementet

$$\Sigma F_x = \iint V_x \rho (\mathbb{V}\cdot\mathbb{n})\, dA + \cancelto{0\ :\ \text{stationärt}}{\frac{d}{dt}} \iiint \rho V_x\, dV \qquad (1)$$

$$\iint V_x \rho (\mathbb{V}\cdot\mathbb{n})\, dA = (V_x \rho (V_x(-1)) A)\Big|_x + (V_x \rho V_x A)\Big|_{x+\Delta x}$$

$$= (V_x^2 \rho \Delta y \Delta z)\Big|_{x+\Delta x} - (V_x^2 \rho \Delta y \Delta z)\Big|_x = \left[\frac{dV_x}{dx}\right] = 0 \qquad (2)$$

$$\left.\begin{aligned} \Sigma F_x = (P\Delta y\Delta z)\Big|_x - (P\Delta y \Delta z)\Big|_{x+\Delta x} + \\ + (\tau \Delta x \Delta z)\Big|_{y+\Delta y} - (\tau\Delta x\Delta z)\Big|_y \end{aligned}\right\} \quad (3)$$

(2) = (3) ger: $(P|_x - P|_{x+\Delta x})\Delta y \Delta z + (\tau|_{y+\Delta y} - \tau|_y)\Delta x \Delta z = 0$

dividera med $\Delta x \Delta y \Delta z$

$$\frac{(P|_x - P|_{x+\Delta x})}{\Delta x} + \frac{(\tau|_{y+\Delta y} - \tau|_y)}{\Delta y} = 0$$

låt Δy & $\Delta x \rightarrow 0$ & använd derivatans definition

$$-\frac{dP}{dx} + \frac{d\tau}{dy} = 0$$

$$\int \frac{dP}{dx}\,dy = \int \frac{d\tau}{dy}\,dy$$ Antag $\frac{dP}{dx}$ konstant

$$\tau = \frac{dP}{dx} \cdot y + C_1$$ symmetri ger $\tau(0) = 0 \Rightarrow C_1 = 0$

$$\tau = \frac{dP}{dx}\,y = \mu \frac{dV_x}{dy}$$

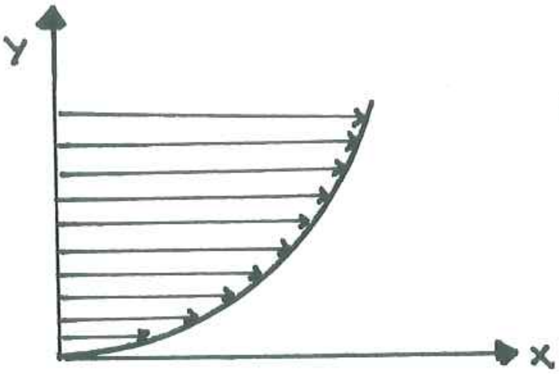
integrera med avseende på y

$$\int \mu \frac{dV_x}{dy}\,dy = \frac{dP}{dx} \int y\,dy \quad \Rightarrow$$

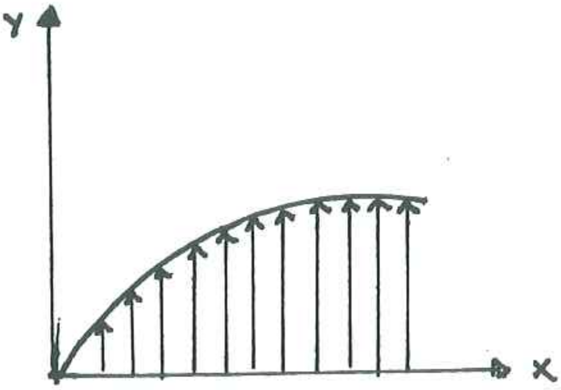
$$V_x = \frac{1}{\mu}\frac{dP}{dx}\int y\,dy = \frac{1}{\mu}\frac{dP}{dx}\frac{y^2}{2} + C_2 \qquad \left(V_x(h) = 0 \right)$$

$$\underline{\underline{V_x = \frac{1}{2\mu}\frac{dP}{dx}\left(y^2 - h^2\right)}}$$ dvs paraboliskt flöde

4.8



$$\frac{dV_x}{dy} \gg \frac{dV_y}{dx}$$

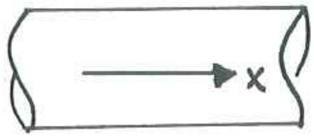


$$\frac{dV_y}{dx} \gg \frac{dV_x}{dy}$$

5.1}

Hagen-Poiseuille : $-\frac{dP}{dx} = \frac{32\mu V_m}{D^2}$

Navier-Stokes : $\rho\frac{DV}{Dt} = \rho g - \nabla P + \mu\nabla^2 V$



endast hastighetskomponent i x-led

cylindriska koordinater:

$$\rho\left(\cancelto{5}{\frac{dV_x}{dt}} + \cancelto{1}{V_r}\frac{dV_x}{dr} + \frac{\cancelto{1}{V_\theta}}{r}\frac{dV_x}{d\theta} + V_x\cancelto{3}{\frac{dV_x}{dx}}\right) =$$

$$= \frac{dP}{dx} + \rho\cancel{g_x} + \mu\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dV_x}{dr}\right) + \frac{1}{r^2}\cdot\cancelto{4}{\frac{d^2V_x}{d\theta^2}} + \cancelto{3}{\frac{d^2V_x}{dx^2}}\right]$$

1: endast strömning i x-led

2: horizontell strömning, tyngdkraften påverkar inte

3: fullt utvecklad strömning, konstant diameter

4: Cylindrisk symmetri

5: stationärt

$\Rightarrow$

$$-\frac{dP}{dx} + \mu \frac{1}{r} \frac{d}{dr}\left(r \cdot \frac{dV_x}{dr} \right) = 0$$

Antag $\frac{dP}{dx}$ är konstant $\circeq$ integrera med avseende på r

$$\frac{dP}{dx} \int r\,dr = \mu \int \frac{d}{dr} \left(r \frac{V_x}{dr} \right) dr \quad \Rightarrow$$

$$\frac{dP}{dx} \frac{r^2}{2} = \mu r \frac{dV_x}{dr} + C_1$$

$$\frac{dV_x\,(r=0)}{dr} = 0 \quad \Rightarrow \quad C_1 = 0 \quad \Rightarrow \quad \frac{dV_x}{dr} = \frac{dP}{dx} \frac{r}{2\mu}$$

integrera med avseende på r:

$$\mu \int \frac{dV_x}{dr} dr = \frac{dP}{dx} \int \frac{r}{2} dr \quad \Rightarrow$$

$$V_x = \frac{1}{\mu} \left(\frac{dP}{dx} \frac{r^2}{4} + C_2 \right)$$

$V_x\,(r=R) = 0$ "no slip" $\Rightarrow$ $C_2 = -\frac{dP}{dx} \frac{R^2}{4\mu}$

$$V_x = \frac{1}{\mu} \frac{dP}{dx} \left(\frac{r^2}{4} - \frac{R^2}{4\mu} \right)$$

$$V_m = \frac{\int V_x \, dA}{\int dA} = \frac{\iint V_x \, r dr d\theta}{\iint r dr d\theta} = \frac{2\pi \int \frac{1}{4\mu} \frac{dP}{dx} (r^2 - R^2) r dr}{2\pi \frac{R^2}{2}}$$

$$V_m = \frac{\frac{dP}{dx}}{L R^2 \mu} \left(\frac{R^4}{4} - \frac{R^2 \cdot R^2}{2} \right) = \frac{dP}{dx} \frac{1}{2R^2\mu} R^4 \left(1/4 - 1/2 \right)$$

$$R = \frac{D}{2} \Rightarrow$$

$$V_m = - \frac{dP}{dx} \cdot \frac{D^2}{32\mu} \quad \Rightarrow$$

$$\underline{\underline{- \frac{dP}{dx} = 32 \mu \frac{V_m}{D^2}}} \quad \blacksquare$$

Äntligen !

5.2)

a,
$$\rho \frac{D\vec{v}}{Dt} = \rho g - \nabla P + \mu \nabla^2 \vec{v}$$

$$\frac{D\vec{v}}{Dt} = \overset{0 \text{ stationärt}}{\cancel{\frac{\partial \vec{v}}{\partial t}}} + \overbrace{V_x \cancel{\frac{\partial \vec{v}}{\partial x}} + V_y \cancel{\frac{\partial \vec{v}}{\partial y}} + V_z \cancel{\frac{\partial \vec{v}}{\partial z}}}^{\approx 0 \text{ små}}$$

krypflöde ⇒ V_x V_y V_z är små jämförelsevis till μ'

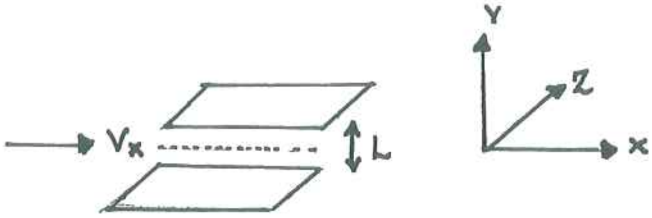
b,
$$\rho \frac{D\vec{v}}{Dt} - \mu \nabla^2 \vec{v} = \rho g - \nabla P$$

$$VL = \underbrace{\rho \left(V_x \frac{\partial \vec{v}}{\partial x} + V_y \frac{\partial \vec{v}}{\partial y} + V_z \frac{\partial \vec{v}}{\partial z} \right)}_{①} - \underbrace{\mu \left(\frac{\partial^2 \vec{v}}{\partial x^2} + \frac{\partial^2 \vec{v}}{\partial y^2} + \frac{\partial^2 \vec{v}}{\partial z^2} \right)}_{②}$$

nära väggarna är 2:a derivatorna mycket
större i ② än förstaderivatorna i ①

① är emot kan den försummas i friströmmen

5.3}



N:S i x-led

$$\rho\left(\overset{①}{\cancel{\frac{\partial V_x}{\partial t}}} + V_x\frac{\partial V_x}{\partial x} + V_y\overset{②}{\cancel{\frac{\partial V_x}{\partial y}}} + V_z\overset{③}{\cancel{\frac{\partial V_x}{\partial z}}}\right) = -\frac{dP}{dx} + \overset{④}{\cancel{\rho g_x}} + \mu\nabla^2\vec{V}$$

① stationärt system

② ingen hastighetskomponent i y-led

③ ———— " ———— z-led

K:E

$$\frac{\partial V_x}{\partial x} + \cancel{\frac{\partial V_y}{\partial y}} + \cancel{\frac{\partial V_z}{\partial z}} = 0$$

$$\Rightarrow \frac{\partial V_x}{\partial x} = 0$$

$$\Rightarrow \frac{\partial^2 V_x}{\partial x^2} = 0$$

$$\Rightarrow \quad \rho V_x\frac{\partial V_x}{\partial x} = -\frac{dP}{dx} + \mu\left(\overset{④}{\cancel{\frac{\partial^2 V_x}{\partial x^2}}} + \frac{\partial^2 V_x}{\partial y^2} + \overset{⑤}{\cancel{\frac{\partial^2 V_x}{\partial z^2}}}\right)$$

④ K:E ger $\frac{\partial V_x}{\partial x} = \frac{\partial^2 V_x}{\partial x^2} = 0$

⑤ oändlig utsträckning i z-led ⇒ ingen hastighetsförändring

$$\Rightarrow \boxed{\frac{dP}{dx} = \mu\frac{\partial^2 V_x}{\partial y^2}}$$

⇒

integrera med avseende på y :

$$\int \frac{dP}{dx}\,dy = \mu \int \frac{\partial^2 V_x}{\partial y^2}\,dy \quad \Rightarrow \left[\text{antag konstant tryckfall}\right]$$

$$\Rightarrow \quad \frac{dP}{dx}\,y + C_1 = \mu \frac{\partial V_x}{\partial y}$$

på grund av symetri är $\frac{\partial V_x}{\partial y} = 0$ vid $y = 0$ (mitt mellan plattorna)

$$\Rightarrow \quad C_1 = 0$$

$$\int \frac{dP}{dx}\frac{y}{\mu}\,dy = \int \frac{\partial V_x}{\partial y}\,dy \quad \Rightarrow \quad V_x = \frac{dP}{dx}\frac{1}{\mu}\left[\frac{y^2}{2} + C_2\right]_0^y$$

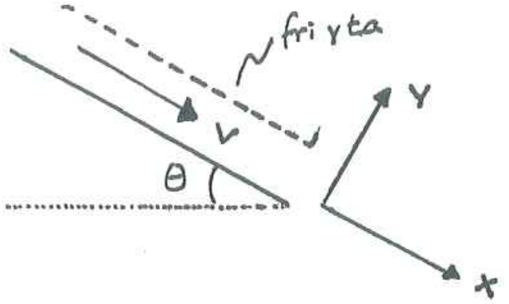
No-slip ger:

$V_x = 0$ vid $y = L/2$ & $y = -L/2$

$$\left.\begin{aligned} V_x\Big|_{y=L/2} &= \frac{dP}{dx}\frac{L^2}{8\mu} + C_2 \\ V_x\Big|_{y=-L/2} &= \frac{dP}{dx}\cdot\left(\frac{-L^2}{8\mu}\right) + C_2 \end{aligned}\right\} \Rightarrow \quad C_2 = \frac{1}{\mu}\frac{dP}{dx}\frac{L^2}{8}$$

$$\Rightarrow \quad \underline{\underline{V_x = \frac{1}{2\mu}\frac{dP}{dx}\left(y^2 + \frac{L^2}{4}\right)}}$$

5..4}



Navier-Stokes i x-led, stationärt

$$\rho g_x - \frac{dP}{dx} + \mu\left(\frac{d^2V_x}{dx^2} + \frac{d^2V_x}{dy^2} + \frac{d^2V_x}{dz^2}\right) = 0$$

$\frac{dV_x}{dz} = 0$ "oändligt" bred platta

$\frac{dV_x}{dx} = 0$ fullt utvecklad strömning

$\frac{dP}{dx} = 0$ fri yta

$$\rho g_x + \mu \frac{d^2V_x}{dy^2} = 0 \qquad g_x = g\sin\theta$$

integrera med avseende på Y

$$\int \frac{d^2V_x}{dy^2}\,dy = -\frac{\rho}{\mu}\int g\sin\theta\,dy$$

6.1)

- V_m L/t
- D L
- ρ M/L^3
- μ M/Lt
- e M

$$\begin{array}{c|ccccc} & V_m & D & \rho & \mu & e \\ \hline M & 0 & 0 & 1 & 1 & 0 \\ L & 1 & 1 & -3 & -1 & 1 \\ T & -1 & 0 & 0 & -1 & 0 \end{array} \sim \begin{array}{ccccc} -1 & 0 & 0 & -1 & 0 \\ 0 & 1 & -3 & -2 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{array}$$

$i = n - r = 2$

Vi väljer kärngrupp som består av $r = 3$ stycken variabler tex ρ v D

$$\begin{cases} \Pi_1 = \rho^a v^b D^c \mu \\ \Pi_2 = \rho^d v^f D^g e \end{cases}$$

Nu gör vi Π_1 & Π_2 dimensionslösa

$$\Pi_1 = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b L^c \frac{M}{Lt}$$

$$= M^{a+1} \cdot L^{-3a+b+c-1} \cdot t^{-b-1} = 1 = M^0 L^0 t^0$$

M: $a+1 = 0$

L: $-3a+b+c-1=0$ $\Rightarrow$ $a=-1$, $b=-1$, $c=-1$

t: $-b-1=0$

$$\Pi_2 = M^d L^{-3d+f+g+1} t^{-f}$$

M: $d=0$

L: $-3d+f+g+1=0$ $\Rightarrow$ $d=0$, $f=0$, $g=-1$

t: $-f=0$

Sätt in a-g i respektive dimensionslösa tal

$$\Pi_1 = \rho^{-1} V^{-1} D^{-1} \mu = \frac{\mu}{\rho V D} = \frac{1}{Re}$$

$$\Pi_2 = D^{-1} e = \frac{e}{D}$$

$$\Rightarrow \underline{\underline{f = f(Re, e/D)}}$$

6.2 $\}$ $\rho = \frac{M}{L^3}$ $\quad \omega = t^{-1}$ $\quad \dot{V} = \frac{L^3}{t}$ $\quad D = L$ $\quad \mu = \frac{M}{Lt}$ $\quad \Delta P = \frac{M}{Lt^2}$

$$\begin{array}{c|cccccc} M & 1 & 0 & 0 & 0 & 1 & 1 \\ L & -3 & 0 & 3 & 1 & -1 & -1 \\ t & 0 & -1 & -1 & 0 & -1 & -2 \end{array} \sim \begin{array}{cccccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & -1 & -1 & 0 & -1 & -2 \\ 0 & 0 & 3 & 1 & 4 & 3 \end{array} \sim \text{rang } 3$$

$i = n - r = 3$

$$\Pi_1 = \rho^a \omega^b D^c \Delta P = \left(\frac{M}{L^3}\right)^a \left(\frac{1}{t}\right)^b L^c \frac{M}{Lt^2}$$

$$\begin{array}{ll} M: & a+1=0 \\ L: & -3a+c-1=0 \\ t: & -b-2=0 \end{array} \Rightarrow \begin{array}{l} a=-1 \\ b=-2 \\ c=-2 \end{array}$$

$$\Rightarrow \rho^{-1}\omega^{-2}D^{-2}\Delta P = \frac{\Delta P}{\rho\omega^2 D^2}$$

$$\Pi_2 = \rho^d \omega^e D^f \dot{V} = \left(\frac{M}{L^3}\right)^d \left(\frac{1}{t}\right)^e L^f \frac{L^3}{t}$$

$$\begin{array}{ll} M: & d=0 \\ L: & -3d+f+3=0 \\ t: & -e-1=0 \end{array} \Rightarrow \begin{array}{l} d=0 \\ e=-1 \\ f=-3 \end{array}$$

$$\Rightarrow \omega^{-1}D^{-3}\dot{V} = \frac{\dot{V}}{\omega D^3}$$

$$\Pi_3 = \rho^g \omega^h D^i \mu = \left(\frac{M}{L^3}\right)^g \left(\frac{1}{t}\right)^h L^i \frac{M}{Lt}$$

$$\begin{aligned} M&: \; g+1=0 \\ L&: \; -3g+i-1=0 \\ t&: \; -h-1=0 \end{aligned} \quad \Longrightarrow \quad \begin{aligned} g&=-1 \\ h&=-1 \\ i&=-2 \end{aligned}$$

$$\Rightarrow \rho^{-1}\omega^{-1}D^{-2}\mu = \frac{\mu}{\rho\omega D^2}$$

$$\Longrightarrow \quad \frac{\Delta P}{\rho\omega^2 D^2} = \underline{\underline{f\left(\frac{\dot{V}}{\omega D^3}, \frac{\mu}{\rho\omega D^2}\right)}}$$

6.3) $Re = \frac{\rho V D}{\mu} = \frac{V D}{\nu}$

a/ $V = 22\ m/s$

$L = 5.8\ m$

$\nu = 15 E^{-6}\ m^2/s$ "luft 20°c"

$\Rightarrow Re_L \approx \underline{\underline{8.51 E^6}}$

b/ $V = 22\ m/s$

$\nu = 15 E^{-6}\ m^2/s$

$D = 0.0064\ m$

$\Rightarrow Re_D \approx \underline{\underline{9.4 E^3}}$

6.4)

$$m = f(D, \rho, \sigma, g)$$

$$D = L \qquad \rho = \frac{M}{L^3} \qquad \sigma = \frac{M}{t^2} \qquad g = \frac{L}{t^2}$$

$$\begin{array}{cccccc} M & 1 & 0 & 1 & 1 & 1 \\ L & 0 & 1 & -3 & 0 & 1 \\ t & 0 & 0 & 0 & -2 & -2 \end{array} \sim \text{rang } 3$$

$i = n - r = 2$ kärngrupp $(D\ \rho\ g)$

$$\pi_1 = \rho^a D^b g^c m = \left(\frac{M}{L^3}\right)^a L^b \left(\frac{L}{t^2}\right)^c M$$

$$\begin{aligned} M&: a + 1 = 0 \\ L&: -3a + b + c = 0 \\ t&: -2c = 0 \end{aligned} \quad \Rightarrow \quad \begin{aligned} a &= -1 \\ b &= -3 \\ c &= 0 \end{aligned}$$

$$\Rightarrow \rho^{-1} D^{-3} m = \frac{m}{\rho D^3}$$

$$\pi_2 = \rho^d D^e g^f \sigma$$

$$\begin{aligned} M&: d + 1 = 0 \\ L&: -3d + e + f = 0 \\ t&: -2f - 2 = 0 \end{aligned} \quad \Rightarrow \quad \begin{aligned} d &= -1 \\ e &= -2 \\ f &= -1 \end{aligned}$$

$$\Rightarrow \rho^{-1} D^{-2} g^{-1} \sigma = \frac{\sigma}{\rho D^2 g}$$

$$\Rightarrow \underline{\underline{\frac{m}{\rho D^3} = f\left(\frac{\sigma}{\rho g D^2}\right)}}$$

7.1}

$D = 0.038\ m$

$\rho = 998.2\ kg/m^3$ "20°c"

$\mu = 1005\ E^{-6}\ Pa\,s$ "20°c"

$Re = \frac{\rho v D}{\mu} = 2300$ vid omslag i rör

$v = \frac{Re\,\mu}{\rho D} = 0.061\ m/s = \underline{\underline{6.1\ cm/s}}$

7.2

$Re = 2E^5$ "se fig 12.4"

$D = 0.042$ m

$\mu = 18.1\,E^{-6}$ Pas

$\rho = 1.189$ kg/m^3

$$\Rightarrow \quad v = \underline{\underline{72.5 \text{ m/s}}}$$

7.3

$v = 30 \text{ m/s}$

$\mu = 18.1 E^{-6} \text{ Pas}$

$Re = 2E^{5}$

$\rho = 1.189 \text{ kg/m}^3$

$\Rightarrow \quad L = 0.1015 \text{ m}.$

7.4}

$C_D = 0.5$ $V_B = 30 m/s$

$A_p = 2,3 m^2$ $\rho = 1.189 kg/m^3$

a, $V_v = 0$

$F = A_P C_D \frac{\rho V_B^2}{2}$

$P = \frac{W}{t}$ $W = F \cdot L \Rightarrow P = \frac{F \cdot L}{t} = F \cdot V_B$

$P = A_P C_D \rho \frac{V_B^3}{2} = \underline{\underline{24.8 \text{ hPa}}}$

b, $v = 30 + 6 = 36 m/s \Rightarrow$

$P = \underline{\underline{35.6 \text{ hPa}}}$

c, $v = 30 - 6 = 24 m/s \Rightarrow$

$P = \underline{\underline{15.8 \text{ hPa}}}$

7.5}

$C_D = 0.5$

$A_p = 2,3\ m^2$

$V = 30\ m/s$

$\rho = 1.189\ kg/m^3$

$F_b = F_p$

$$\Rightarrow \left(A_p C_D \frac{\rho v^2}{2}\right)_b = \left(A_p C_D \frac{\rho v^2}{2}\right)_p$$

bil $A_p C_D = 0.5 \cdot 2,3 = 1.15$

antag $Re = 2000$

$$\Rightarrow C_D = 1.01$$

$$\Rightarrow A_p = \frac{D^2 \pi}{4}$$

$$\Rightarrow 1.15 = 1.01 \frac{D^2 \pi}{4}$$

$$\Rightarrow D = \underline{\underline{1.2\ m}}$$

test av Re

$$Re = \frac{\rho v D}{\mu} = 2,4E^6 > 2000 \Rightarrow \text{ok!}$$

7.6)

a, laminärt flöde vid v = 30 m/s

använd Blasius ekvation för laminärt flöde

$$\delta = \frac{5x}{\sqrt{\frac{vx}{\nu}}} \qquad \text{där } \nu = 1.805 E^{-5}\ m^2/s$$

$$= \frac{5x}{\sqrt{Re_x}} = \frac{5\sqrt{x}}{\sqrt{\frac{30}{\nu}}}$$

$$= \underline{\underline{0.003541421\sqrt{x}}} \qquad \text{plotta } \delta \text{ mot } x$$

b, $$\delta = \frac{0.376\,x}{Re_x^{1/5}} = \underline{\underline{0.020667\,x^{4/5}}}$$

7.7 $V_\infty = 63$ m/s på 1500 meters höjd

a, helt laminärt:

$$F = A\, C_f\, \frac{\rho V_\infty^2}{2}$$

$$A = 2 \cdot 2.1 \cdot 12.2 = 51.24\ \text{m}^2$$

$$\rho = 1.058\ \text{kg/m}^3$$

$$\mu = 1.742\ E^{-5}\ \text{Pas}$$

$$R_e = \frac{v \cdot L \rho}{\mu} = 8.035\ E^{6}$$

$$C_{fL} = \frac{1.328}{\sqrt{R_e}} = 4.685\ E^{-4}$$

$$\Rightarrow \quad F = \underline{\underline{50.4\ \text{N}}}$$

b, helt turbulent

$$C_{fx} = \frac{0.0576}{{R_{ex}}^{1/5}}$$

medelfriktionsfaktorn för hela vingen fås genom integration över hela vingens bredd

$$C_{f_L} \cdot L = \int_0^L C_{fx}\, dx = \frac{0.0576}{\left(\frac{V_\infty \rho}{\mu}\right)^{1/5}} \int x^{-1/5}\, dx$$

OBS division med L

$$= \frac{5}{4} \frac{0.0576}{\left(\frac{V_\infty \rho}{\mu}\right)^{1/5}} \cdot L^{4/5} = \boxed{\frac{0.072}{Re_L^{\,1/5}}}$$

$$\Rightarrow \quad F = \underline{\underline{322.2\ N}}$$

c) laminärt = turbulent, omslag vid $Re_x = 2 \cdot 10^5$

$\Rightarrow x_{KRIT} = 0.0523\ m$

Laminära delen

$$C_f = \frac{1.328}{\sqrt{Re_L}} = 2.969\, E^{-3} \quad \Rightarrow \quad F_L = 7.952\ N$$

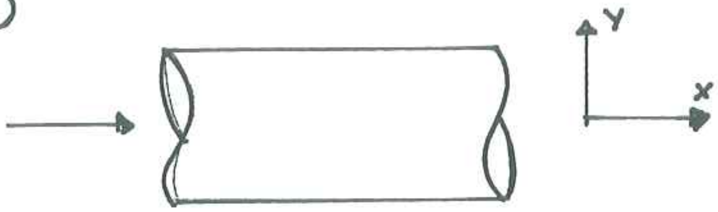
Turbulenta delen

$$C_f\,(x_2 - x_1) = \int C_{fx}\, dx = \frac{0.072}{\left(\frac{V_\infty \rho}{\mu}\right)^{1/5}} \left[x_2^{4/5} - x_1^{4/5} \right]$$

$$C_f = 2.911\, E^{-3} \quad \Rightarrow \quad F_T = 305.4\ N$$

$$F_{TOT} = \underline{\underline{313\ N}}$$

8.1)



$D = 0.006\ m$

$\dot{V} = 10.5E^{-6}\ m^3/s$

$L = 15\ m$

$\nu = 7.4E^{-6}\ m^2/s$

$\rho = 913\ kg/m^3$

$$V = \frac{\dot{V}}{A} = \frac{\dot{V}4}{D^2\pi}$$

$$Re = \frac{v\rho D}{\mu} = \frac{vD}{\nu} = 301$$

laminärt flöde ⇒ vi kan använda H.P

$$\Rightarrow -\frac{dP}{dx} = \frac{32\mu v}{D^2}$$

$$\Rightarrow \Delta P = \frac{32\rho\nu}{D^2} \cdot \frac{4\dot{V}}{D^2\pi}$$

$$= 128\frac{\rho\nu\dot{V}L}{\pi D^4}$$

$$= 33.45\ kPa$$

8.2

$$Y_1 + \frac{V_1^2}{2g} + \frac{P_1}{\rho} = Y_2 + \frac{V_2^2}{2g} + \frac{P_2}{\rho}$$

a,

$$\left.\begin{aligned} \Delta P &= 0 \\ V_1 &\approx 0 \\ -\Delta Y &= h + L \end{aligned}\right\} \quad \frac{V_2^2}{2g} = h + L$$

$$\Rightarrow \quad V_2 = \sqrt{2g\,(h+L)}$$

b,

$$\frac{V_2^2}{2g} = h + L + \sum F$$

$$\sum F = 2 f_f \frac{L v^2}{Dg} + \frac{K v^2}{2g} \qquad \Rightarrow$$

$$v^2 + 4 f_f \frac{L v^2}{D} + K v^2 = 2g\,(h+L) \qquad \Rightarrow$$

$$v = \sqrt{\frac{2g\,(h+L)}{1 + K + \frac{4Lf_f}{D}}}$$

$$\lim_{L \to \infty} = \sqrt{\frac{gD}{2f_f}}$$

8.3}

$D = 0.0025\ m$

$L = 0.76\ m$

$\nu = 7.4E^{-6}\ m^2/s$

$\rho = 913\ kg/m^3$

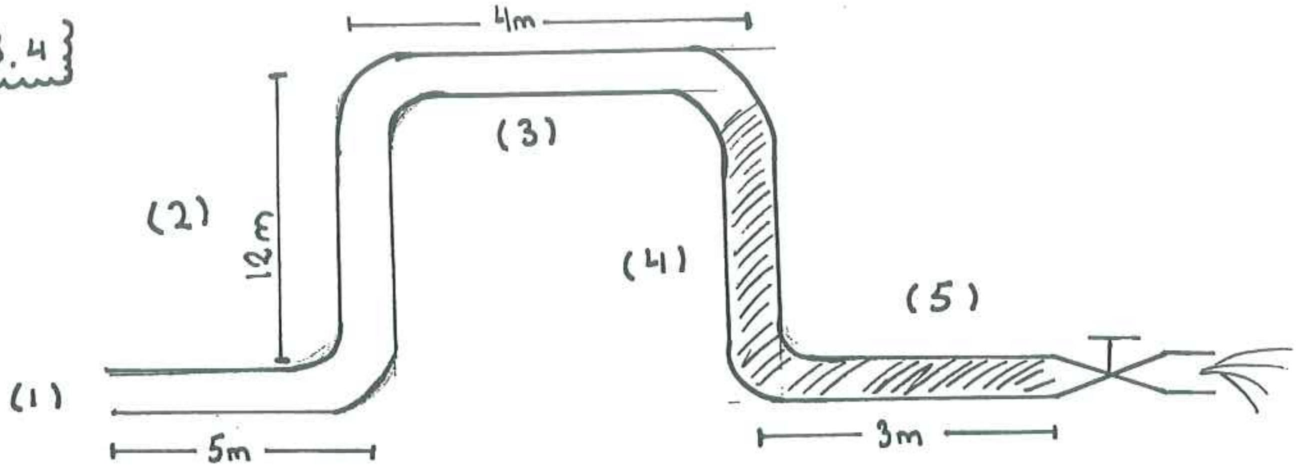
$\Delta P = 103\ kPa$

$$-\frac{dP}{dx} = 32\frac{\mu v}{D^2}$$

$$\Rightarrow V = -\frac{dP\, D^2}{dx\, 32\, \nu \rho}$$

$$= \underline{\underline{3.92\ m/s}}$$

8.4



$$\Delta P_f = \Delta P_{slät} + \Delta P_{skrovlig} + \Delta P_{krökar} + \Delta P_{ventil}$$

friktionsförluster:

$$\Delta P = 2g f_f \frac{L}{D} v^2 = \left[Q = v \cdot A \right] = 32 \rho f_f \frac{L Q^2}{\pi^2 D^5}$$

$$\Delta P_{slät} = \Delta P_1 + \Delta P_2 + \Delta P_3 = 32 \rho f_f \frac{Q^2}{\pi^2 D^5} (L_1 + L_2 + L_3)$$

$Re = 31700 > 2300$ turbulent: f_f fås ur diagram 14.1

$$\left. \begin{aligned} e/D &= 3E^{-5} \\ Re &= 31700 \end{aligned} \right\} \Rightarrow f_f = 0.0058$$

$$\Rightarrow \Delta P_{slät} = 30\,852 \ Pa$$

$\Delta P_{skrovlig} = \Delta P_4 + \Delta P_5$

$$\left.\begin{array}{l} Re = 31\,700 \\ e/D = 3E^{-3} \end{array}\right\} \Rightarrow f_f = 0.00735$$

$$\Rightarrow \Delta P_{skrovlig} = 27\,927 \text{ Pa}$$

Engångsförluster:

$$\Delta P = K\frac{\rho v^2}{2} = \left[Q = v \cdot A \right] = K\frac{8\rho Q^2}{\pi^2 D^4}$$

$K = 0.5$ antal krökar = 4

$$\Rightarrow \Delta P_{krökar} = 2533 \text{ Pa}$$

$$K = 7 \Rightarrow \Delta P_{ventil} = 8866 \text{ Pa}$$

$$\Rightarrow \Delta P_f = \underline{\underline{70.2 \text{ kPa}}}$$

9.1

$$\left.\begin{array}{ll} \rho_s = 1600 \text{ kg/m}^3 & \rho = 1.12 \text{ kg/m}^3 \\ D_s = 0.005 \text{ mm} & \mu = 1.813 \text{E}^{-5} \text{ Pas} \end{array}\right\} \text{luft } 20^\circ\text{C}$$

$$W_{max} = \left(\frac{4(\rho_s - \rho)}{3 \rho C_D} g D_P \right)^{1/2} \qquad (1)$$

$$C_D = C_D(W_{max})$$

$$C_D = \frac{24}{Re} \quad \text{lågt Reynolds antas} \quad Re < 2 \qquad (3)$$

$$Re = \frac{\rho W_{max} D_s}{\mu} \qquad (2)$$

Iterera !

Gissa $C_D \overset{(1)}{\Rightarrow} W_{max} \overset{(2)}{\Rightarrow} Re$... kontrollera $Re < 2$

$\overset{(3)}{\Rightarrow} C_D$

$\Rightarrow W_{max} = 4.3$ m/h

9.2)

$\rho = 1000\ kg/m^3$ $\rho_{öga} = 4000\ kg/m^3$

$H = 11'000\ m$ $\mu = 1519\ E^{-6}\ Pas$

$D = 0.03m$

$T = 5°C$

Accelerationstiden antas vara kort o försumbar

$$\Rightarrow \quad t = \frac{H}{V_{max}}$$

$$V_{max} = \left(\frac{4 \Delta \rho}{3 \rho C_D} g D_p \right)^{1/2} \qquad (1)$$

$$Re = \frac{\rho D}{\mu} \cdot V_{max} \qquad (2)$$

$C_D = C_D(Re)$ enligt diagram 12.4

Iteration

Gissa $C_D = 0.45 \overset{(1)}{\Rightarrow} V_{max} = 1.62\ m/s \overset{(2)}{\Rightarrow} Re = 32'000$

$\overset{(3)}{\Rightarrow} C_D = 0.45$ OK!

$V_{max} = 1.62 m/s \Rightarrow t = 6790\,s = \underline{1h\ 53\,min}$

9.3}

$\rho = 1000 \ kg/m^3$

$H = 200 \ m$

$\rho_{Pb} = 11'300 \ kg/m^3$

$T = 5°C$

$t = 62.5 \ s$

$\mu = 1519 E^{-6} \ Pa s$

Accellerationstiden försummas

$$v = \frac{200}{62.5} = 3.2 \ m/s$$

$$v = \left(\frac{4 \Delta \rho \, g \, D_p}{3 \rho \, C_D} \right)^{1/2}$$

$$Re = \frac{\rho \, v \, D_p}{\mu}$$

Iterera!

Gissa $C_D = 0.48 \Rightarrow D_p = 0.03645$

$\Rightarrow Re = 73\,351 \Rightarrow C_D = 0.48$ OK!

$$\Rightarrow D_p = \underline{\underline{3.6 \ cm}}$$

10.1

$D_p = 0.02\,m$ $\rho_l = 930\ kg/m^3$

$V = 0.3\ m/s$ $\rho_{Pb} = 11\,300\ kg/m^3$

$$V = \left(\frac{4 \Delta \rho\, g \underline{D}_p}{3 \rho_l C_D} \right)^{1/2} \Rightarrow C_D = 32.4$$

$$C_D = \frac{24}{Re} = \frac{\mu 24}{V \rho D_p} \Rightarrow \mu = 7.53 \text{ Pas}$$

10.2)

$$P_1 = \rho_{HG} \cdot g \cdot (H_2 + H_3) + 1\,atm$$

$$P_2 = P_1 - \rho_f g H_2$$

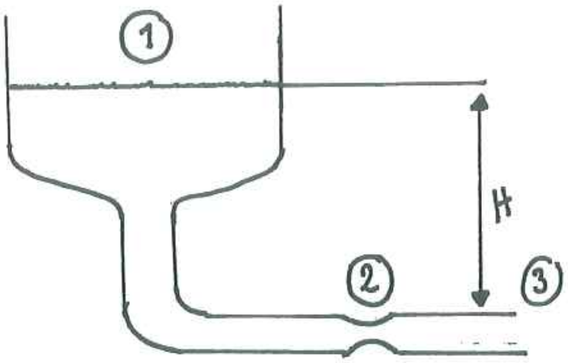
$$= g(\rho_{HG}(H_2 + H_3) - \rho_f H_2) + 1\,atm$$

$$A = P_2 - \rho_v g H_1$$

$$= g(\rho_{HG}(H_2 + H_3) - \rho_f H_2 - \rho_v H_1) + 1\,atm$$

$$= \underline{\underline{39.1\,kPa + 1\,atm}}$$

10.3}



$H = 2.4 m$

$A_2 = 0.003 m^2$

$A_3 = 0.005 m^2$

a, $P_1 + \frac{\rho v_1^2}{2} + \rho g h_1 = \frac{\rho v_3^2}{2} + \rho g h_3 + P_3$

$P_1 = P_3 = 1 atm$

$\Rightarrow \quad \rho g H = \frac{\rho}{2}(V_3^2 - V_1^2)$

$\Rightarrow \quad \frac{2 \Delta P}{\rho} = V_3^2 \qquad ty \; V_1 = 0$

$V_3 = \sqrt{\frac{2 \Delta P}{\rho}}$

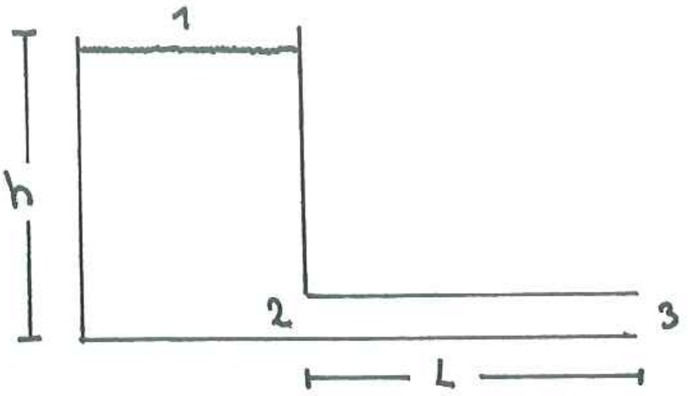
$Q = V_3 \cdot A_3 = \underline{\underline{0.034}} \, m^3/s$

b, $V_2 A_2 = V_3 A_3 \quad \Rightarrow \quad V_2 = 11.44 \, m/s$

$P_2 + \frac{\rho V_2^2}{2} = P_3 + \frac{\rho V_3^2}{2} \quad \Rightarrow \quad P_2 = P_3 + \frac{\rho}{2}(V_3^2 - V_2^2)$

$\Rightarrow \quad P_2 = \underline{\underline{1 atm - 41.8 \, kPa}}$

10.4



h = 8 cm

L = 55 cm

d = 0.18 cm

$$\Delta P = \frac{128 \mu L Q}{\pi D^4}$$ från uppgift 4.5

$$P_2 - P_1 = \rho g h$$

$$P_2 = P_{atm} + \rho g h \quad ; \quad P_2 - P_3 = P_2 - P_{atm}$$

$$\Delta P = P_2 - P_3 = \rho g h$$

$$\frac{\mu}{\rho} = \nu = \left(\frac{128 \, L \, Q}{g h \pi D^4} \right)^{-1} = \underline{\underline{1.348 E^{-6} \, m^2/s}}$$

10.5} Ställ upp N:s i z-led med cylindriskakoordinater

$$\rho\left(\cancelto{1}{\frac{\partial V_z}{\partial t}} + V_r\cancelto{4}{\frac{\partial V_z}{\partial r}} + \frac{V_\theta}{r}\cancelto{4}{\frac{\partial V_z}{\partial \theta}} + \cancelto{2}{\frac{\partial V_z}{\partial z}}\right) =$$

$$-\frac{\partial P}{\partial z} + \rho\cancelto{3}{g_z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right) + \frac{1}{r^2}\cancelto{4}{\frac{\partial^2 V_z}{\partial \theta^2}} + \cancelto{2}{\frac{\partial^2 V_z}{\partial z^2}}\right]$$

1, stationärt

2, fullt utvecklad strömning

3, Verkar endast i y-led

4, symetri ger $V_\theta = V_r = 0 \Rightarrow \frac{\partial}{\partial \theta} = 0$

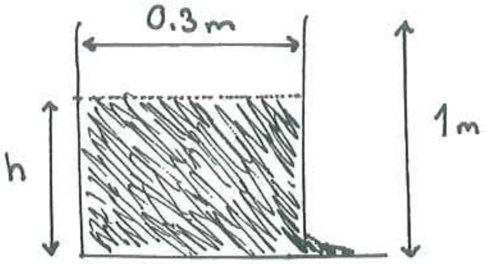
$$\Rightarrow \frac{dP}{dz} = \frac{\mu}{r}\frac{\partial}{\partial r}$$

10.6)

$A = 0.008\ m^2$
$\eta = 0.15$

$\rho = 1000\ kg/m^3$
$d = 0.3\ m$

$H = 0.003\ m$



$$\phi = \frac{(d+0.006)^2\pi}{4} - \frac{d^2\pi}{4} = 0.0029$$

$$m_c = \phi\,\rho\,h_t = 23.2\ kg$$

$$m_t = m_c + \frac{d^2\pi}{4}\cdot 0.003\,\rho = 24.89\ kg$$

$$F_f = \eta M g = \eta g\left(m_t + \frac{d^2\pi}{4}h\right)$$

$$F_x = \iint_{C.S} v\rho(v\cdot n)\,dA = v^2\rho A \qquad v = \sqrt{2gh}$$

$$\Rightarrow \quad F_x = 2ghA\rho$$

$$F_x = F_f$$

$$2\rho g h A = \eta g\left(m_t + \frac{d^2\pi}{4}h\rho_v\right)$$

$$\Rightarrow \quad \frac{m_t}{h} = \frac{2gA}{\eta g} - \frac{d^2\pi}{4}\rho_v$$

$$\Rightarrow \quad h = \frac{m(\eta g - 4)}{\rho 2gA - d^2\pi\rho_v} = \underline{\underline{0.73\ m}}$$

10.7) $\frac{P}{V}$ = konstant

$$P = \frac{ML^2}{t^3} \qquad \rho = \frac{M}{L^3} \qquad \omega = t^{-1} \qquad D = L$$

$$\begin{array}{ccccc} M & 1 & 1 & 0 & 0 \\ L & 2 & 3 & 0 & 1 \\ t & -3 & 0 & -1 & 0 \end{array} \sim \begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 3 & -1 & 0 \end{array} \sim \begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{array} \sim \text{rang } 3$$

$i = n - r = 1$ dimensionslöst tal

$$\Pi_1 = \rho^a \omega^b D^c P = \left(\frac{M}{L^3}\right)^a \left(\frac{1}{t}\right)^b L^c \frac{ML^2}{t^3}$$

$$\begin{aligned} M&: a + 1 = 0 \\ L&: -3a + c + 2 = 0 \\ t&: -b - 3 = 0 \end{aligned} \implies \begin{aligned} a &= -1 \\ b &= -3 \\ c &= -5 \end{aligned}$$

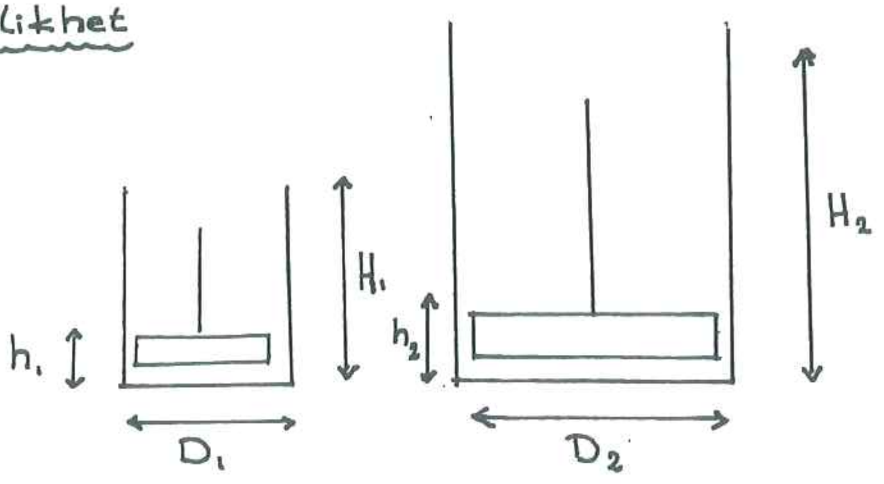
$\Rightarrow \rho^{-1}\omega^{-3}D^{-5}P = \frac{P}{\rho\omega^3 D^5}$ skall vara konstant

Geometrisk likhet

$$\frac{D_1}{H_1} = \frac{D_2}{H_2}$$

$$\frac{D_1}{d_1} = \frac{D_2}{d_2}$$

$$\frac{D_1}{h_1} = \frac{D_2}{h_2}$$



$$\left.\begin{aligned} V_1 &= \frac{\pi d_1^2}{4} H_1 \\ V_2 &= 3V_1 \end{aligned}\right\} \quad \frac{\pi d_2^2 H_2}{4} = \frac{3 d_1^2 \pi H_1}{4} \Rightarrow d_2^2 = \frac{3 d_1^2 H_1}{H_2}$$

Geometrisk Likhet: $\frac{H_1}{H_2} = \frac{d_1}{d_2} \Rightarrow \underline{\underline{d_2 = d_1 \sqrt[3]{3}}}$

$\frac{P}{V}$ konstant $\frac{P_1}{P_2} = \frac{1}{3}$:

$$\omega_2^3 = \frac{P_2 \omega_1^3 D_1^5}{P_1 D_2^5} = \frac{3\omega_1^3 D_1^5}{D_2^5} = 3\,\sqrt[-3]{3}^{\,5}\,\omega_1^3$$

$$= \underline{\underline{3^{-2/9}\,\omega_1}}$$

ω skall minska med $3^{-2/9}$ ≗ d skall ökas med $\sqrt[3]{3}$

10.8)

$$\frac{F}{A_p} = C_D \frac{\rho v^2}{2}$$

$$Re = \frac{\rho v D}{\mu} = 8868$$

$$\Rightarrow C_D = 1.2$$

$\rho = 1.189 \, kg/m^3$

$\mu = 18.1 E^{-6} \, Pas$

$L = 1m$

$D = 0.005m$

$v = 27m/s$

$$A_p = 0.005 \cdot 1$$

$$\Rightarrow F = \frac{C_D A_p \rho v^2}{2} = \underline{\underline{2.6 \, N}}$$

10.9)

$$v = \frac{\dot{V}}{A} = \frac{4\dot{V}}{D^2\pi}$$

$$Re = \frac{vD}{\nu} = \frac{4\dot{V}}{\nu\pi D}$$

$Re >> 2300$

Alltså turbulent

$L = 8000$ m

$D = 5$ m

$\dot{V} = 90\ m^3/s$

$\Delta h = 668$ m

$\nu = 0.995E^{-6}\ m/s$

$\rho = 998\ kg/m^3$

figur 14.2 $\Rightarrow e/D = 0.0006$

$\Rightarrow f_f = 0.0044$ genom figur 14.1

$$\frac{\Delta P_f}{\rho} = 2 f_f \frac{L}{D} v^2 \quad \Rightarrow \quad \Delta P_f = 2.952E^5$$

$$\frac{P_0}{\rho} + g h_0 + \frac{v_0^2}{2} = \frac{P_1}{\rho} + g h_1 + \frac{v_1^2}{2} + \frac{\Delta P_f}{\rho}$$

$$\Rightarrow P_1 = P_0 + \rho\left(g\Delta h - \frac{v^2}{2}\right) - \Delta P_f$$

$$= 6.24E^6 + atm$$

$$= \underline{\underline{62.6\ atm}}$$

10.10}

$$\dot{V} = \frac{0.3}{60} = 0.005\ m^3/s$$

$$Re = \frac{vD}{\nu} = \frac{\dot{V}D}{A\nu}$$

$$= \frac{4\dot{V}}{\nu\pi D} = \frac{6398}{D}$$

$$\Delta P_f = 2\rho f_f \frac{L}{D} v^2$$

$$= 2\rho f_f \frac{L}{D}\left(\frac{4\dot{V}}{\pi D^2}\right)^2 = \frac{0.067 f_f}{D^5}$$

$$\Rightarrow D = 0.1808\, f_f^{1/5}$$

iterera med hjälp av tabell 14.1

$\Rightarrow$ $\underline{\underline{6.1\ cm}}$

$\dot{V} = 0.3\ m^3/min$

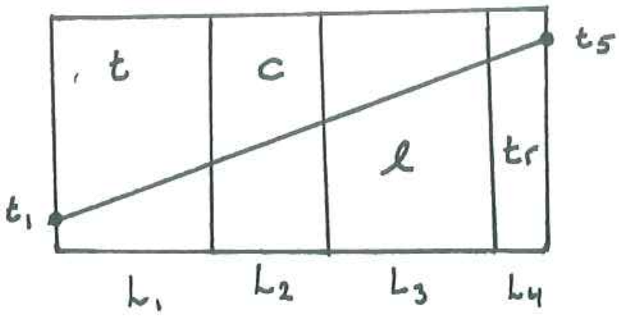
$L = 75\ m$

$\Delta P = 31372.25\ Pa$

$\rho = 998\ kg/m^3$

$\nu = 0.995E^{-6}\ m^2/s$

11.1



$k_t = 0.66$ W/mK
$k_c = 0{,}048$ -11-
$k_l = 0.026$ -11-
$k_{tr} = 0.21$ -11-
$k_g = 0{,}043$ -11-

$$q_x \cdot A^{-1} = -k\frac{dT}{dx}$$

$$\Rightarrow q_x A^{-1}\int_0^L dx = -k\int_{t_1}^{t_5} dT$$

$$\Rightarrow q_x = \frac{kA\Delta t}{L}$$

ingen ackumulation:

$$\frac{k_t A_1}{L_1}(t_1-t_2) = \frac{k_c A_2}{L_2}(t_2-t_3) = \frac{k_l A_3}{L_3}(t_3-t_4) = \frac{k_{tr} A_4}{L_4}(t_4-t_5)$$

$$(t_1-t_5) = q_x\frac{L_1}{k_t A_1} + \ldots + q_x\frac{L_4}{k_{tr} A_4}$$

Vi har även $A_1 = A_2 = A_3 = A_4$

$$\Rightarrow t_1 - t_5 = \frac{q_x}{A}\left(\frac{L_1}{k_t} + \ldots + \frac{L_4}{k_{tr}}\right)$$

$$\Rightarrow q_x A^{-1} = -6.4\ \mathrm{W/m^2}$$

11.2

$$qA^{-1} = -h_5(T_4 - T_5) = -h_1(T_1 - T_i) = -\frac{k}{L_1}(T_2 - T_1)$$

$$\Rightarrow qA^{-1} = \frac{T_i - T_4}{\frac{1}{h_5} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \frac{L_4}{k_4} + \frac{1}{h_1}}$$

a, $qA^{-1} = 6.2$

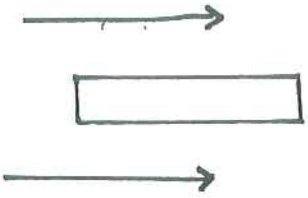
b, $qA^{-1} = 44$

c, $qA^{-1} = 9.41$

$h_1 = 39.7\ W/m^2K$

$h_2 = 11.4\ W/m^2K$

11.3



$T_0 = 850\ K$

$k = 1.35\ W/mK$

$h = 5\ W/m^2K$

$\frac{dT}{dx} = -1500\ K/m$

$q_{ledning} = q_{konvektion}$

$$\Rightarrow \quad h\Delta T = -k\frac{dT}{dx}$$

$$\Rightarrow \quad T_{luft} = \underline{\underline{445\ K}}$$

11.4)

$T_0 = 850 K$

$k = 1.35 \; W/mK$

$h = 5 \; W/m^2K$

$\frac{dT}{dx} = -1500 \; K/m$

$\sigma = 5.676 \, E^{-8} \; W/m^2K$

$q_l = q_c + q_r$

$$\Rightarrow \; -k\frac{dT}{dx} = h\Delta T + \sigma T^4$$

$$\Rightarrow \; hT_l + \sigma T_l = hT_0 + \sigma T_0^4 - 1500k = 31854.07$$

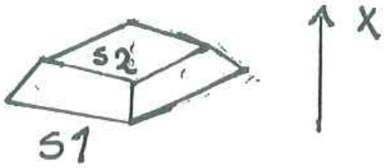
1/ Använd miniräknarens ekvationslösare

$\Rightarrow T_l = 835.6 K$

2/ Gissa T_l & se om du får 31854.07

$\Rightarrow T_l = 835.6 K$

11:5



s1 = 0.1 m

s2 = 0.05 m

h = 0.15 m

T_1 = 300 K

T_2 = 600 K

k = 0.173 W/mK

$$q_x A^{-1} = -k \frac{dT}{dx}$$

$$q_x \int A^{-1} dx = -k \int dT = -k\Delta T \qquad (1)$$

$A = s^2$ där s ökar linjärt från s2 till s1

$$\Rightarrow \quad s = ax + b$$

$$\Rightarrow \quad \left\{ \begin{matrix} s1 = a \cdot 0 + b \\ s2 = ah + b \end{matrix} \right\} \Rightarrow S2 = ah + s1$$

$$\Rightarrow \quad a = \frac{\Delta s}{h} = -1/3$$

$\Rightarrow \quad s = -1/3\, x + 0.1$ insätts i (1) & integrera

$$\Rightarrow \quad q_x = \underline{\underline{-1.74 \text{ W}}}$$

11.6)

$S_2 = 0.05$ m

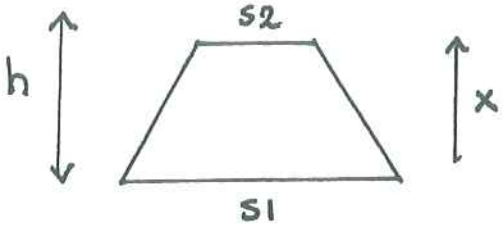
$S_1 = 0.1$ m

$T_2 = 300$ K

$T_1 = 600$ K

$h = 0.15$ m

$k = 0.173$ W/mK



$$q_x \int A^{-1} dx = -k \int dt$$

$A = s^2$ där $s = -\frac{1}{3}x + 0.1$

$$\Rightarrow \quad q_x = \underline{\underline{1.73 \text{ W}}}$$

För mer förklaringar, se uppgift 10.5

11.7}

$T_2 = 600 K$

$T_1 = 300 K$

$s_2 = 0.05 m$

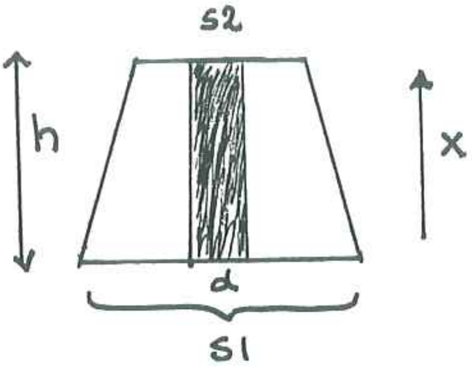
$s_1 = 0.1 m$

$h = 0.15 m$

$d = 0.01905 m$

$k_a = 0.173 W/mK$

$k_b = 40 W/mK$



$$q = q_{asbest} - q_{hål} + q_{bult}$$

$q_{asbest} = 1.73 W$ från uppgift 10.6

$$q_{bult} = -\frac{kA\Delta T}{h}$$

$$A = \frac{d^2\pi}{4} = 2.85E^{-4} m^2$$

$$\Rightarrow q_{bult} = -22.8 W$$

$$q_{hål} = -\frac{kA\Delta T}{h} = -0.0986$$

$$\Rightarrow q_x = \underline{\underline{-24.4 W}}$$

12.1 }

$$\underbrace{\cancel{\frac{\partial T}{\partial t}}}_{0} = \alpha \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \underbrace{\cancel{\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}}}_{0} + \underbrace{\cancel{\frac{\partial^2 T}{\partial z^2}}}_{0} \right]$$

stationärt   bara i radiell led

$$0 = \alpha \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} \right] \Rightarrow 0 = \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r} \right)$$

b) $\begin{cases} T = T_i \quad \text{då } r = r_i \\ T = T_o \quad \text{då } r = r_o \end{cases}$

$$C_1 = r\frac{\partial T}{\partial r} \quad \Rightarrow \quad T = C_1 \ln(r) + C_2$$

$$T_i = C_1 \ln r_i + C_2$$

$$T_o = C_1 \ln r_o + C_2$$

$$\Rightarrow T_i - T_o = C_1 \ln r_i - C_1 \ln r_o = C_1 \ln\left(\frac{r_i}{r_o}\right)$$

$$\Rightarrow C_1 = \frac{T_i - T_o}{\ln \frac{r_i}{r_o}}$$

$$C_2 = T_i - C_1 \ln r_i = T_i - \frac{T_i - T_o}{\ln \frac{r_i}{r_o}} \ln r_i$$

$$\Rightarrow \underline{\underline{\frac{T - T_i}{T_i - T_o} = \frac{\ln r/r_i}{\ln r_i/r_o}}}$$

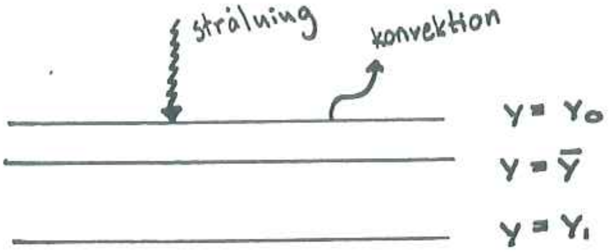
c) $q_r A^{-1} = -k \frac{dT}{dr}$

$$\frac{dT}{dr} = \frac{c_1}{r} = \frac{T_i - T_o}{r \ln r_i/r_o}$$

$$A = 2\pi r \cdot L$$

$$\Rightarrow \quad q_r = -k \, 2\pi L \cdot \frac{T_i - T_o}{\ln r_i/r_o}$$

122 }



$h = 12 \; W/m^2K$

$k_p = 2.42 \; W/mK$

$k_{Al} = 218 \; W/mK$

a,

$T(\bar{y}) = 325 \; K \qquad T_{luft} = 295 \; K$

$q_{strålning} = q_{ledning} + q_{konvektion}$

$$(qA^{-1})_l = \frac{T(\bar{y}) - T(y_0)}{\frac{1}{h} + \frac{l_{Al}}{k_{Al}}} = 359.01 \; W/m^2$$

För att beräkna $(qA^{-1})_k$ behövs $T(y_0)$

$$(qA^{-1})_l = -k \frac{(T(\bar{y}) - T(y_0))}{0.025} \Rightarrow T(y_0) = 328.7 \; K$$

$$(qA^{-1})_k = h\Delta T = 404.5 \; W/m^2$$

$$\Rightarrow q_{strålning} = \underline{\underline{763.5 \; W/m^2}}$$

b, strålning på Al-plattan

$\Rightarrow$ stega uppifrån

$(qA^{-1})_l = 320.3 \ W/m^2$

Nu behövs $T(y_1)$

$(qA^{-1})_l = -k \dfrac{T(\bar{y}) - T(y_1)}{0.05} \Rightarrow T(y_1) = 325.07$

$(qA^{-1})_k = h \Delta T = 360.9 \ W/m^2$

$\Rightarrow q_{strålning} = \underline{\underline{681.2 \ W/m^2}}$

12.3 }

$$\nabla^2 T = \frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

$$\int \frac{d}{dr}\left(r\frac{dT}{dr}\right) dr = r\frac{dT}{dr} = C_1$$

$$\int \frac{dT}{dr}\, dr = \int \frac{C_1}{r}\, dr$$

$$\left.\begin{aligned} T_o &= C_1 \ln r_o + C_2 \\ T_i &= C_1 \ln r_i + C_2 \end{aligned}\right\} \Rightarrow C_1 = \frac{T_o - T_i}{\ln\frac{r_o}{r_i}} \qquad C_2 = T_i - \frac{T_o - T_i}{\ln\frac{r_o}{r_i}} \cdot \ln r_i$$

$$q = -kA\frac{dT}{dr} = -2k\pi r \cdot L \cdot \frac{C_1}{r} = -k2\pi L \cdot \frac{(T_o - T_i)}{\ln\frac{r_o}{r_i}} =$$

$$= k\,\frac{r_o - r_i}{r_o - r_i}\,\frac{2\pi L}{\ln\frac{r_o}{r_i}}\,(T_i - T_o) = \frac{k}{r_o - r_i}\,\frac{2\pi L(r_o - r_i)}{\ln(r_o/r_i)}\,(T_i - T_o)$$

$$\Rightarrow \bar{A} = \frac{2\pi L(r_o - r_i)}{\ln(r_o/r_i)}$$

b/ istället för $\overline{A}$ används $\widetilde{A} = \pi L(r_o + r_i)$

$$q = \frac{k\pi(r_o + r_i)\,L\,\Delta T}{r_o - r_i} = \frac{k\pi\left(r_o/r_i + 1\right)\cdot L \cdot \Delta T}{r_o/r_i - 1}$$

$$A = \left(\frac{r_o/r_i + 1}{r_o/r_i - 1}\right) \text{ istället för } \frac{2}{\ln(r_o/r_i)} = B$$

$\frac{r_o}{r_i} = 1.5$ $\quad A = 5$ $\quad B = 4.9326$ $\quad \frac{A}{B} = 1.0137$

$\frac{r_o}{r_i} = 3$ $\quad A = 2$ $\quad B = 1.8205$ $\quad \frac{A}{B} = 1.0986$

$\frac{r_o}{r_i} = 5$ $\quad A = 1.5$ $\quad B = 1.2427$ $\quad \frac{A}{B} = 1.2071$

12.4 }

Värmeledning i en ihålig sfär

sfäriska koordinater ger : (Laplace)

$$\nabla^2 T = \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) + \frac{1}{r^2 \sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dT}{d\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{d^2T}{d\varphi}$$

sfärisk symmetri ⇒ endast r-beroende

$$\int \frac{d}{dr}\left(r^2\frac{dT}{dr}\right) dr = 0 \Rightarrow \frac{dT}{dr} = \frac{C_1}{r^2}$$ se 11.3 för detaljer

$$\int \frac{dT}{dr}\,dr = \int \frac{C_1}{r^2}\,dr \Rightarrow T = -\frac{C_1}{r} + C_2$$

$$\left.\begin{aligned} T(r_o) &= T_o \\ T(r_i) &= T_i \end{aligned}\right\} \Rightarrow C_1 = \frac{T_i - T_o}{r_i - r_o}\cdot r_i r_o \qquad C_2 = \frac{T_i r_i - T_o r_o}{r_i - r_o}$$

$$q = -kA\nabla T = -k4\pi r^2\frac{C_1}{r^2} = \frac{-k}{r_i - r_o}(T_i - T_o)\,4\pi r_i r_o$$

$$= \frac{k}{r_o - r_i}\Delta T \bar{A} \qquad \text{där} \qquad \bar{A} = 4\pi r_i r_o$$

b, Aritmetisk medelarea : $\overline{A}_A = \frac{A_i + A_o}{2} = 2\pi(r_i^2 + r_o^2)$

Geometrisk medelarea från uppgift a, $\overline{A}_G = 4\pi r_i r_o$

$$q = \frac{k\overline{A}}{r_o - r_i} \Delta T$$

procentuella felet : $q_A / q_G = B$

$\frac{r_o}{r_i} = 1.5 \Rightarrow B = 1.0833$

$\frac{r_o}{r_i} = 3 \Rightarrow B = 1.667$

$\frac{r_o}{r_i} = 5 \Rightarrow B = 2.6$

12.5 }

$d_y = 0.5\,m$	$k = 0.002\ W/mK$
$\delta = 25\,mm$	$r_{N_2} = 200\ kJ/kg$
$T_i = 77\ K$	$\rho = 804\ kg/m^3$
$T_y = 298.15\,K$	$h_y = 18\ W/m^2$

$$q_r = \frac{\Delta T}{\Sigma R} = \frac{k\bar{A}}{r_o - r_i}\,\Delta T$$ där $\bar{A} = 4\pi r_o r_i$ för ledning

$$\Rightarrow \quad R_{LED} = \frac{r_o - r_i}{4\pi k r_o r_i}$$

$$R_{KON} = \frac{1}{4\pi r_o^2 h_y}$$ ty $\frac{\Delta T}{R} = hA\Delta T$

$$\Rightarrow \quad q_r = \frac{\Delta T}{\frac{r_o - r_i}{4\pi k r_o r_i} + \frac{1}{4\pi r_o^2 h_y}} = 15.21\ J/s$$

$$\dot{m} = q/r_{N_2} = 7.605\,E^{-5}\ kg/s = \underline{\underline{0.274}}\ kg/h$$

131}

$C_p = 3.35 \ kJ/kg\,K$ $\qquad$ $T_0 = 5°C$

$\rho = 880 \ kg/m^3$ $\qquad$ $T_v = 100°C$

$k = 0.5 \ W/mK$ $\qquad$ $h = 90 \ W/m^2K$

$B_i = \frac{h(V/A)}{k} = 0.9$ $\qquad$ $B_i > 0.1 \Rightarrow$ diagram lösning

$$\left.\begin{aligned} Y &= \frac{T_v - T}{T_v - T_0} = 0.21 \\ X &= \frac{\alpha t}{x_1^2} \\ x_1 &= 0.01 \\ n &= 0 \\ m &= \frac{k}{h x_1} = 0.556 \end{aligned}\right\} \quad X = 0.8$$

$\Rightarrow \ t = 472 s = 7.9 \ min$

13.2 } $T_o = 204°C$ $c_p = 125\ J/kgK$

$t = 15\ s$ $k = 35\ W/mK$

$h = 230\ W/m^2K$ $\rho = 11340\ kg/m^3$

$B_i = 0{,}0056 < 0.1$ ⇒ analytisk lösning

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-B_i F_o}$$

$$F_o = \frac{\alpha t}{(V/A)^2} = 512{,}6$$

$$\Rightarrow \frac{T - T_\infty}{T_o - T_\infty} = 0{,}057$$

$$\Rightarrow T = 41.7°C \approx \underline{\underline{42°C}}$$

13.3

$D = 0.15 \text{ m}$

$T_0 = 1090 °C$

$T_\infty = -18 °C$

$h = 91 \text{ W/m}^2\text{K}$

$k = 40 \text{ W/mK}$

$C_P = 0.42 \text{ kJ/kgK}$

$\rho = 7400 \text{ kg/m}^3$

$$\alpha = \frac{k}{\rho C_p}$$

$$Bi = \frac{h(V/A)}{k} = 0.057 < 0.1$$

$$\Rightarrow \frac{T - T_\infty}{T_0 - T_\infty} = e^{-Bi\,Fo} = 0.3014$$

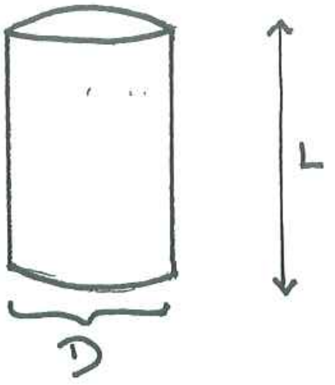
$$\Rightarrow Fo = \frac{-\ln(0.3014)}{Bi} = 21.03 = \frac{\alpha t}{(V/A)^2}$$

$$\Rightarrow t = \frac{Fo\,(V/A)^2 \rho C_p}{k}$$

tänk på enheter !

$$t = \underline{\underline{17.9 \text{ min}}}$$

13.4



$L = 0.61\ m$

$D = 0.076\ m$

$T_0 = 21°c$

$T_\infty = 540°c$

$h = 23\ W/m^2K$

$k = 367\ W/m^2K$

$T = 260\ °c$

$$B_i = \frac{h(V/A)}{k}$$

$$\left.\begin{array}{l} V = \frac{D^2\pi L}{4} \\ A = \pi D L \end{array}\right\} \Rightarrow B_i = 0.0012 < 0.1$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{-B_i F_0} \Rightarrow F_0 = 514.26 = \frac{\alpha t}{(V/A)^2}$$

$$t = \frac{F_0 (V/A)^2}{\alpha}$$

slå upp α för koppar i appendix

$$\Rightarrow t = \underline{\underline{28.9\ min}}$$

13.6}

$D = 0{,}028\,m$ $\quad\rho = 1059\ kg/m^3$

$T_\infty = 180°c$ $\quad k = 0.4\ W/mK$

$T_0 = 20°c$ $\quad C_p = 3200\ J/kgK$

$T = 60°c$ $\quad h = 28\ W/m^2K$

$Bi = 0.33 > 0.1$ ⇒ diagramlösning appendix F

$$Y = \frac{T_\infty - T}{T_\infty - T_0} = 0{,}75$$

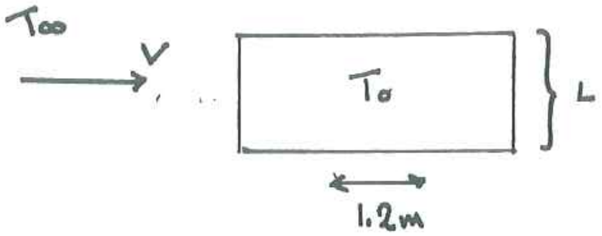
$$n = \frac{x}{x_1} = 0$$ ty $x=0$ centrum

$$m = \frac{k}{hx_1} = 1.02$$

$$\Rightarrow X = 0.22 = \frac{\alpha t}{x_1^2}$$

$$\Rightarrow t = \frac{0.22\, r^2 \rho C_p}{k} = \underline{\underline{6\ min}}$$

14.1



$T_\infty = 38^\circ C$

$P = 1 atm$

$v = 3 m/s$

$L = 0.15 m$

$T_0 = 93^\circ C$

$M = 28 g/mol$

a, $\delta = \frac{5x}{\sqrt{Re_x}} = \left[Re_x = 183673 \right] =$

$= \underline{\underline{1.4 cm}}$

b, $\delta_t = \frac{\delta}{Pr^{1/3}}$ $\quad Pr = 0.713$ finns tabelerat

$\Rightarrow \delta_t = \underline{\underline{1.6 cm}}$

c, $C_{fx} = \frac{0.664}{\sqrt{Re_x}} = \underline{\underline{0.00155}}$

d, $C_{fL} = \frac{1.328}{\sqrt{Re_L}} = \underline{\underline{0.003}}$

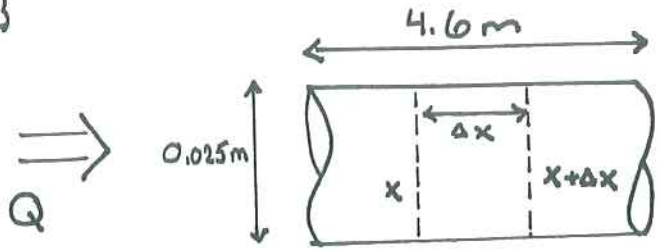
e, $h_x = 0.332 \frac{k}{x} Re_x^{1/2} P_r^{1/3} = \underline{\underline{3.04\ W/m^2K}}$

f, $h = \frac{k \cdot 0.664 P_r^{1/3} \sqrt{Re_x}}{x} = \underline{\underline{6.1\ W/m^2K}}$

g, $F = C_{fL} \rho A \frac{v^2}{2} = \underline{\underline{2.54\, \bar{E}^{3}\, N}}$

h, $q = hA\Delta T = \underline{\underline{60W}}$

14.2}



$T_v = 16°C$

$T_{yta} = 150°C$

$$\left\{\begin{matrix}\text{värme in}\\\text{genom flöde}\end{matrix}\right\} + \left\{\begin{matrix}\text{värme in genom}\\\text{konvektion}\end{matrix}\right\} = \left\{\begin{matrix}\text{värme ut}\\\text{genom flöde}\end{matrix}\right\}$$

det vill säga $q_1 + q_2 = q_3$

$$q_1 = V \cdot A \cdot \rho \cdot C_p \cdot T|_x$$

$$q_2 = h \pi D \cdot \Delta x \; (T_{yta} - T)$$

$$q_3 = V \cdot A \cdot \rho \cdot C_p \cdot T|_{x+\Delta x}$$

$$A = \frac{\pi D^2}{4}$$

$$\Rightarrow \; V A \rho C_p \left(T|_{x+\Delta x} - T|_x\right) = h \pi D \Delta x \, (T_s - T)$$

$$\Rightarrow \; \frac{V D \rho C_p}{4} \cdot \frac{dT}{dx} = h(T_s - T)$$

$$\Rightarrow \; \int_{T_0}^{T_L} \frac{dT}{T_s - T} = \frac{4h}{V D \rho C_p} \int_0^L dx \;\Rightarrow\; -\ln\left(\frac{T_L - T_s}{T_0 - T_s}\right) = \frac{4h}{V D \rho C_p} \cdot L$$

$$\frac{4L}{D} \cdot \frac{h}{V \rho C_p} = \frac{4L}{D} \cdot \underset{}{\overset{\text{stanton}}{St}} \;\Rightarrow\; -\ln\left(\frac{T_L - T_s}{T_0 - T_s}\right) = St \cdot \frac{4L}{D}$$

Reynolds analogi: $St = \frac{C_f}{2}$

$C_f = C_f(Re)$ $Re = \frac{vD}{\nu}$ $\nu = \nu(T)$

Måste ta reda på om flödet är turbulent!

ν minskar vid högre T. Den lägsta temperaturen utflödet kan ha är $T_0 = 16°C$

$\nu(T_0) = 1.1E^{-6} \Rightarrow Re = 8.8E^{4} > 2300$

flödet kommer alltid vara turbulent!

För turbulent strömning i slätt rör ger:

$$\frac{1}{\sqrt{C_f}} = 4 \log_{10}\left\{Re\, C_f\right\} - 0.4 \quad (14\text{-}12)$$

$Re = Re(T) \Rightarrow$ iteration krävs

Gissa $T_L = 100°C$

$$\Rightarrow T_{filmmedel} = \frac{\frac{100+16}{2} + 150}{2} = 104°C$$

$$\Rightarrow \left\{\nu = 0.277E^{-6}\right\} \quad Re = 3.5E^{5}$$

$$\Rightarrow C_f = 0.0037$$

⇒ $St = 0,00185$

⇒ $T_L = 115.7°C \gg 100°C$ ej OK

Gissa $T_L = 116°C$ ⇒ [...] $T_L = 116$ är OK!

Den totala värmeöverföringen:

$$q_{Re} = \dot{m} C_p \Delta T = \rho Q C_p \Delta T$$

ρ ≗ C_p tas vid fluidens medeltemperatur = 66°C

$$\Rightarrow \quad q_{Re} = \underline{\underline{778\ kW}}$$

Colburns analogi: $St = \frac{C_f}{2} \cdot P_r^{-2/3}$

Gissa T_L ⇒ St ⇒ T_L osv... $T_L = 98.5$

$$q_{cb} = \rho Q C_p \Delta T$$

ρ ≗ C_p tas vid fluidens medeltemperatur = 57°C

$$\Rightarrow \quad q_{cb} = \underline{\underline{644\ kW}}$$

Reynold överskattas värmeöverföringen

14.3} Se uppgift 13.2 för detaljer

$$-\ln\left(\frac{T_L - T_s}{T_0 - T_s}\right) = S_t \cdot \frac{4L}{D}$$

$$\Rightarrow \quad T_L = 150 - 136\, e^{-736\, St}$$

Reynold: $C_f/2 = St$

Colburn: $C_f/_2 \, P_r^{-2/3} = St$

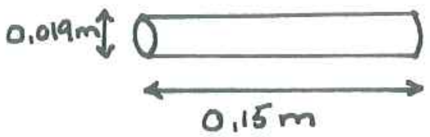
$$\text{Gissa } T_L \Rightarrow T_{\text{filmmedel}} \Rightarrow \underset{(P_r)}{\nu} \Rightarrow Re \Rightarrow C_f \Rightarrow St \Rightarrow T_L$$

Reynold : $T_L = 146°C \quad \Rightarrow \quad q_{Re} = 295 \ W$

Colburn : $T_L = 148°C \quad \Rightarrow \quad q_{cb} = 300 \ W$

$(q = v \cdot A \cdot \rho \cdot C_p \, \Delta T)$

14.4



$T_\infty = 35^\circ C$

$q = 750\ W$

$$q = h \cdot A \cdot \Delta T = h \cdot \pi D L (T_s - T_\infty)$$

$$\Rightarrow T_s = T_\infty + q / h\pi DL = 35 + \frac{12564.9}{Nu \cdot K}$$

$$Nu = Nu(Gr, Pr)$$

a, $$Ra = Gr \cdot Pr = \frac{\beta \overset{\sim 0{,}15}{L^3} g (T_s - T_\infty) \cdot Pr}{\nu^2}$$ data vid $T_f = \frac{T_\infty + T_s}{2}$

Gissa $T_s \Rightarrow \beta, Pr, \nu^2 \Rightarrow Ra > 10^9 \Rightarrow$

$$Nu = \left[0.825 + \frac{0.387 \cdot Ra^{1/6}}{(1 + (0.492/Pr)^{9/16})^{8/27}} \right] \Rightarrow$$

$T_s = 92^\circ C$

Kriterie: $\frac{D}{L} = 0.127 \ngeq \frac{35}{Gr^{1/4}} = 0.13$ men tillräckligt nära!

Lite fusk är tillåtet

b, $Nu = C \, Ra_D^{\,n}$ $\qquad C = 0{,}125 \qquad n = 0{,}333$

$$Ra_D = Gr_D \cdot Pr = \frac{\beta g D^3 \Delta T}{\nu^2} Pr$$

Gissa $T_s \Rightarrow \beta, Pr, \nu \Rightarrow Ra_D \Rightarrow Nu \Rightarrow T_s$

$T_s = 95°C$

14.5 } $T_{so} = 360\ K$ $k = 0.2\ W/mK$

$T = 320\ K$ $\rho = 1120\ kg/m^3$

$T_{\infty} = 295\ K$ $C_p = 1020\ J/kgK$

Data tas vid T_f för fluiden

$$T_f = \frac{\frac{T_{so} + T_{\infty}}{2} + \frac{T_s + T_{\infty}}{2}}{2} = 317.5\ K$$

$Nu_D = 2 + 0.43 Ra_D^{1/4}$ Naturlig konvektion vid sfär

$Ra_D = Gr \cdot Pr$

$$\left.\begin{array}{l} Pr = 0.703625 \\ k = 2.75919 \\ \frac{\beta g \rho^2}{\mu^2} = 1.0358\ E^8 \end{array}\right\} Ra_D = 327967\ E^4 \cdot d^3$$

$$Nu = \frac{hd}{k}$$

$$Bi = \frac{h\,(V/A)}{k}$$

uppgift a & b ger diagramlösning

uppgift c ger analytisk lösning

b, $\alpha = 0.01$ m   Bi > 0.1 grafisk lösning

$$\left.\begin{aligned} Y &= 0.385 \\ n &= 1 \\ m &= 2.76 \end{aligned}\right\} \Rightarrow X = 0.95 \Rightarrow t = \underline{\underline{135.6\ s}}$$

c, $\alpha = 0.002$ m   Bi < 0.1 analytisk lösning

$$\frac{T - T_\infty}{T_{s0} - T_\infty} = e^{-Bi\,Fo} \Rightarrow t = \underline{\underline{8.9\ s}}$$

14.6} $q = 630\ W/m^2$ $\qquad$ $T_\infty = 10°C$

$T_s = 66°C$ $\qquad$ $T_f = 38°C$

$v = 6.1\ m/s$

Förlust: $q = A h \Delta T$

Omslag till turbulent flöde sker vid $Re = 2E^5$

$$Re = \frac{v \cdot L_L}{\nu} \Rightarrow [\nu \text{ tas vid } T_f] \Rightarrow L_L = 0.557\ m$$

laminärt : $$Nu = \frac{h \cdot L_L}{k} = 0.664\, Re^{1/2} P_r^{1/3} = 263.66$$

turbulent : $$Nu_x = 0.0288\, Re_x^{4/5} P_r^{1/3} \Rightarrow$$

$$Nu_{L_T} = \left(\int_{L_L}^{L_T} dx \right)^{-1} \cdot \int 0.0288\, Re_x^{4/5} P_r^{1/3}\, dx$$

$$= \frac{1}{L_T - L_L} \cdot 0.0288 \cdot \frac{5}{4} P_r^{1/3} \left[Re \right]_{L_L}^{L_T}$$

$$A \cdot \bar{h} = A_L h_L + A_T h_T = L_L \cdot 6 \cdot h_L + (L_T - L_L) \cdot 6 \cdot h_T =$$

$$\left[263.66 + 0.036 P_r^{1/3} \left(Re_{L_T}^{4/5} - Re_{L_L}^{4/5} \right) \right] \cdot k \cdot 6 = 92.7 \cdot 6$$

$$q = \frac{92.7}{6} (T_s - T_\infty) = 864.9\ W/m^2$$

$$\Rightarrow \frac{864.9}{630} = \underline{\underline{1.37}}$$

14.7}

$q = 630\ W/m^2$ $T_\infty = 10°C$

$T_s = 66°C$ $T_f = 38°C$

$L = \frac{yta}{Omkrets} = \frac{6}{4} = 1.5$

$Ra_L = Gr\,Pr = \frac{\beta g \rho^2}{\mu^2} L^3 \Delta T = 1.546\ E^{10}$

$Nu = 0.14\ Ra_L^{1/3}$ (20-8)

$\Rightarrow\ h = 6.299\ W/m^2K$

$q_K A^{-1} = h \Delta T = 353\ W/m^2$

$q_K/q = \underline{\underline{0.56}}$

15.1 } $P = 2.76$ MPa $\phi = 21.9$ cm

$T = 427°C$ $\delta = 2.06$ cm

$\dot{m} = 1.25$ kg/s $\mu = 24.25 \, E^{-5}$ Pas

$$D = \phi - 2\delta = 17.78 \text{ cm}$$

$$\left. \begin{array}{l} Re = \dfrac{vD\rho}{\mu} \\ \\ v = \dfrac{\dot{m}}{\rho A} \end{array} \right\} Re = 369130 > 2300 \quad \text{turbulent}$$

$$Nu_D = 0.023 \, Re_D^{0.8} \, Pr^n \qquad (20-26)$$

ångan kyls $\Rightarrow$ $n = 0.3$

$$Nu_D = \frac{hD}{k} = 653.8 \quad \Rightarrow \quad h = \underline{\underline{186 \ W/m^2K}}$$

15.2

$$qA^{-1} = h\Delta T$$

$$Nu_D = \frac{hD}{k} = 2 + 0.43\, Ra_D^{1/4} \qquad (20-12) \qquad 1 < Ra_D < E^5$$

Materialdata tas vid filmtemperaturen

$$T_f = \frac{T_0 - T_\infty}{2} = 800 \text{ K}$$

$$Ra_D = \frac{\beta g D^3 Pr}{\nu^2} \Delta T = 9.94 E^3$$

$$\Rightarrow \quad Nu_D = 6.294 \quad \Rightarrow \quad h = 18.17 \ W/m^2K$$

$$qA^{-1} = 18170 \approx \underline{\underline{18200 \ W/m^2}}$$

15.3}

$\phi = 0.15\,m$ $\qquad$ $T_s = 1300\,K$

$T_\infty = 300K$ $\qquad$ $v = 150\,m/s$

$$T_f = \frac{1300 + 300}{2} = 800$$

$$Re_D = \frac{vD}{\nu} = 2.7E^5$$

$$Nu_D = 2 + 0.6\,Re_D^{1/2}\,Pr^{1/3} = 279.3$$

$$\Rightarrow \quad h = 107.52\ W/m^2K$$

$$q A^{-1} = h\Delta T = 107.52\ W/m^2$$

15.4

vatten	rökgaser
$\dot{m} = 13.889$ kg/s	$\dot{m} = 13.889$ kg/s
$T_{in} = 60°C$	$T_0 = 427°C$
$T_{ut} = 93\,°C$	$C_p = 1.005$ kJ/kgK
$T_m = 76.5\,°C$	$h = 68$ W/m²K

a, $(\dot{m}\, C_p\, \Delta T)_v = (\dot{m}\, C_p\, \Delta T)_g$

$\Rightarrow T_{gut} = 289°C$

b, $$q/A = U\, \Delta T_{lm} = U \frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)} = 18932.87\ W/m^2$$

$q = \dot{m}\, C_p\, \Delta T = 1920.88$ kW

$$\Rightarrow A = \underline{\underline{101.46\ m^2}}$$

$$\frac{dV_x}{dy} = -\frac{\rho}{\mu} g \sin\theta \; y + C_1$$

$$\left.\frac{dV_x}{dy}\right|_{y=L} = 0 \Rightarrow C_1 = \frac{\rho g \sin\theta \, L}{\mu}$$

$$\frac{dV_x}{dy} = -\frac{\rho g \sin\theta}{\mu} y + \frac{\rho g \sin\theta \cdot L}{\mu}$$

integrera med avseende på y:

$$V_x = -\frac{\rho g \sin\theta}{2\mu} y^2 + \frac{\rho g \sin\theta \cdot L}{\mu} \cdot y + C_2$$

$$\left.V_x\right|_{y=0} = 0 \quad \text{"no slip"} \quad \Rightarrow \quad C_2 = 0$$

$$\underline{\underline{V_x = \frac{\rho g L^2}{\mu} \left[\frac{y}{L} - \left(\frac{y}{L}\right)^2 \frac{1}{2} \right] \sin\theta}}$$

15.5}

$\phi_y = 0.02\,m$ $T_y = -15°C$

$\phi_i = 0.015\,m$ $T_i = 5°C$

$\delta = 0.01\,m$ $Q = 0.05E^{-3}\,m^3/s$

$h_i = 1600\,W/m^2K$ $k_s = 35\,W/mK$

$h_y = 40\,W/m^2K$ $k_i = 0.05\,W/mK$

b)

$$q = \dot{m} c_p \Delta T + \dot{m}\Delta H_c = 17.754\,kW$$

$$qA^{-1} = -k\frac{dT}{dr} \Rightarrow q/_{2\pi dL}\int_{r_i}^{r_y} r^{-1}dr = -k(T_y - T_i)$$

$$\Rightarrow q/_{2\pi dL}\ln\left(\frac{r_y}{r_i}\right) = -k\Delta T$$

$$q/_{2\pi r_i dL} = h_i\,(T_v - T_i)$$

$$q/_{2\pi r_y dL} = h_y\,(T_y - T_L)$$

$$\Rightarrow q = \frac{T_v - T_L}{\frac{\ln(r_y/r_i)}{2\pi k dL} + \frac{1}{2\pi r_i h_i dL} + \frac{1}{2\pi r_y h_y dL}} \qquad (*)$$

kan jämföras med $q = UA\Delta T_{lm}$

$$(*) \Rightarrow u \cdot A = 2.4\, dL$$

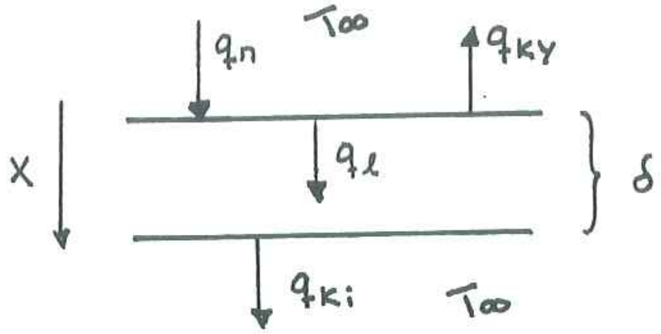
$$\Delta T_{lm} = -17.38 \quad \Rightarrow \quad \underline{\underline{dL = 25\, m}}$$

a)

$$u \cdot A = \frac{1}{\frac{\ln(r_y/r_i)}{2\pi K dL} + \frac{1}{2\pi r_i h_i dL} + \frac{1}{2\pi r_y h_y dL} + \frac{\ln(r_y/\delta)}{2\pi K_i dL}}$$

$$q = uA\, \Delta T_{lm} \quad \Rightarrow \quad dL = \underline{\underline{146.9\, m}}$$

15.6



$q_n = 500\ \mathrm{W/m^2}$

$\delta = 1\ \mathrm{dm}$

$k = 1\ \mathrm{W/mK}$

$L = 5\ \mathrm{m}$

$h_i = 10\ \mathrm{W/mK}$

$T_\infty = 20^\circ\mathrm{C}$

Insida:

$$q_\ell = q_{ki} \Rightarrow h_i(T_{si} - T_\infty) = \frac{k}{\delta}(T_{sy} - T_{si})$$

$$\Rightarrow T_{si} = \frac{k/\delta\, T_{sy} + h_i T_\infty}{h_i + k/\delta} \qquad (1)$$

Utsida

$$q_n = q_{ky} + q_\ell = h_y(T_{sy} - T_\infty) + k/\delta\,(T_{sy} - T_{si}) \qquad (2)$$

$$(1)\ i\ (2) \Rightarrow \frac{q_n + \left(h_y + \frac{k/\delta \cdot h_i}{h_i + k/\delta}\right) T_\infty}{h_y + k/\delta - \frac{(k/\delta)^2}{h_i + k/\delta}} = T_{sy}$$

$$\Rightarrow \quad T_{sy} = \frac{500 + (h_y + 5)\,293}{h_y + 5} \qquad (*)$$

Måste ha h_y. Naturlig konvektion horizontell platta

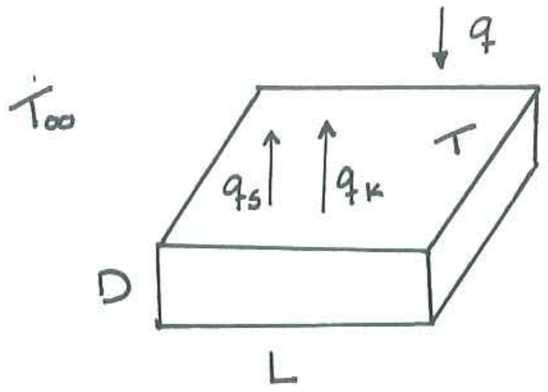
$$Nu = 0.14\, Ra_L^{1/3}$$

$$Ra_L = Gr\,Pr = \frac{g\,\beta\,L^3\,Pr\,\Delta T}{\nu^2} \qquad T_f = \frac{T_{sy} + T_{si}}{2}$$

Gissa $T_{sy} \Rightarrow T_f \Rightarrow \beta\ Pr\ \nu^2 \Rightarrow Ra_L \Rightarrow Nu \Rightarrow h_y \Rightarrow \overline{T_{sy}}$

$$\Rightarrow \quad T_{sy} = \underline{\underline{66.5\,^\circ C}}$$

16.1}



$L = 0.61$ m

$D = 0.035$ m

$q = 117$ J/s

$\sigma = 5.6704 E^{-8}$ W/m^2K^4

$h = 22.7$ W/m^2K

$T_\infty = 305$ K

$$q_s A^{-1} = \sigma (T^4 - T_\infty^4)$$

$$q_K A^{-1} = h(T - T_\infty)$$

$A = L^2$ försummar sidoeffekter

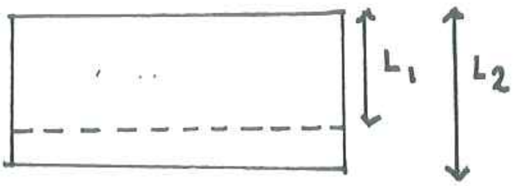
$$117 = A\sigma(T^4 - T_\infty^4) + Ah(T - T_\infty)$$

Inmatning i miniräknarens ekvationslösare ger:

$$T = \underline{\underline{42.5°C}}$$

Alternativ lösning är iteration

16.2



$T_\infty = -10°C$

$\Delta H_f = 334\ kJ/kg$

$\rho = 910\ kg/m^3$

$L_1 = 0.05\ m$

$L_2 = 0.15\ m$

$k = 2.1\ W/mk$

$T_1 = 0°C$

Bilden är lite fel

Men skit i det ☺

Linjär temperaturprofil

$$\underbrace{\Delta H_f \rho \Delta L}_{(1)} = \underbrace{q A^{-1} \Delta t}_{(2)}$$

(1) Den värme som frigörs vid isbildning

(2) Den värme som transporteras genom ledning

$$q A^{-1} = -k \frac{\partial T}{\partial L} \quad \Rightarrow \quad q A^{-1} = -\frac{k(T_\infty - T_1)}{L_2}$$

$$\Rightarrow \Delta H_f \rho \Delta L = -k \frac{(T_\infty - T_1)}{L_2} \Delta t$$

Låt Δ → 0 & integrera

$$\Delta H_f \rho \int_{L_1}^{L_2} L\, dL = -k(T_\infty - T_1) \int_0^t dt$$

$$\Rightarrow t = \frac{\left[\frac{L^2}{2}\right]_{L_1}^{L_2} \Delta H_f \rho}{-k(T_\infty - T_1)} = \underline{\underline{40.2\ h}}$$

16.3} Se uppgift 13.2 för detaljer

$$T_L = 150 - 57e^{-736St}$$

Reynold: $St = C_f/2$

Colburn: $St = C_f/2 \; Pr^{-2/3}$

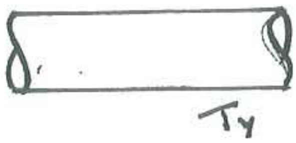
Gissa $T_L \Rightarrow T_{filmmedel} \Rightarrow \nu \, Pr \Rightarrow Re \Rightarrow C_f \Rightarrow St \Rightarrow T_L$

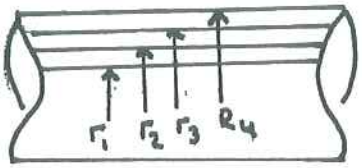
$q = v \cdot A \cdot \rho \cdot c_p \, \Delta T$

Reynold: $T_L = 138°C \Rightarrow q_{Re} = 108$ kW

Colburn: $T_L = 150°C \Rightarrow q_{Cb} = 137$ kW

16.4}





$r_1 = 0.02 m$

$r_2 = 0.0219 m$

$k_m = 0.066 W/mK$

2.54 cm tjockt lager

$T_i = 4.5°c$

$T_y = 38°c$

$k_g = 0.038 W/mK$

2.54 cm tjockt lager

$k_s = 40 W/mK$

$$q A^{-1} = -k \frac{dT}{dr}$$

$$A = 2\pi r L$$

Cylindriska koordinater:

$$q \int \frac{1}{r} dr = -2\pi L k \int dt$$

$$q \ln\left(\frac{r_B}{r_A}\right) = -2\pi L K (T_B - T_A)$$

$$q = \frac{2\pi L K (T_A - T_B)}{\ln\left(\frac{r_B}{r_A}\right)}$$

$$q = \frac{\Delta T}{\Sigma R}$$ Definition

b/ $h_i = 568 \ W/m^2K$ $\qquad h_y = 28.4 \ W/m^2K$

$$qA^{-1} = h\Delta T \Rightarrow q = \frac{\Delta T}{\frac{1}{hA}} \Rightarrow R = \frac{1}{hA}$$

$$R_{TOT} = \frac{4.2652}{L} + \frac{1}{h_i 2\pi r_3 L} + \frac{1}{h_y 2\pi r_4 L}.$$

$$\Rightarrow LR_{TOT} = 4.3563 \ mK/W$$

$$qL^{-1} = \frac{\Delta T}{L \cdot R_{TOT}} = 7.69 \ W/m$$

$$q \cdot A_y^{-1} = q(2\pi r_y L)^{-1} = -16.835 \ W/m^2$$

16.5}

$T_\infty = 5°C$

$v = 0.5\ m/s$

$h = 100\ W/m^2K$

$d_y = 0{,}035\ m$

$d_i = 0{,}025\ m$

$k = 0.6\ W/mK$

$$U \cdot A = \frac{2\pi \cdot L}{\frac{\ln(d_y/d_i)}{k} + \frac{2}{h\, d_y}} = 593.8\ W/K$$

$q = UA\,\Delta T_{lm}$

materialdata tas vid medeltemperetur

$q = \dot{m}\, C_p\, \Delta T$

Gissa $T \Rightarrow q \Rightarrow T$

$\Rightarrow T = 76°C$

17.1

	mol	molmassa
metan	93.5%	16.042
etan	4.6%	30.068
propan	1.2%	44.094
koldioxid	0.7%	44.01

a,

$$\begin{aligned} 0.935 \cdot 16.042 &= 14.99927 \\ 0.046 \cdot 30.068 &= 1.383128 \\ 0.012 \cdot 44.094 &= 0.529128 \\ 0.007 \cdot 44.01 &= \underline{0.30807} \\ & 17.219596 \end{aligned}$$

$$\frac{1.383128}{17.219596} = \underline{\underline{8.03 \text{ vikts\%}}}$$

b, $\underline{\underline{17.22 \text{ g/mol}}}$ se uppgift a

c, $PV = nRT = \frac{m}{M} RT$

$$\rho = \frac{m}{V} = \frac{PM}{RT} = 1400.76 \approx \underline{\underline{1.4 \text{ kg/m}^3}}$$

d, $$y_A = \frac{P_A}{RT} \Big/ \frac{P}{RT} = \frac{P_A}{P}$$

$$\Rightarrow P_A = y_A \cdot P = 1.31 E^5 \text{ Pa}$$

e, $$\omega_A = \frac{\rho_A}{\rho} \approx 18000 \text{ ppm}$$

17.2

a/ $\mathbf{N}_A + \mathbf{N}_B = c\mathbf{V}$

$\mathbf{N}_A$: molära fluxet relativt stationära koordinater

$\mathbf{V}$: molbaserad medelhastighet för blandningen

$\mathbf{N}_A = c\mathbf{v}_A$

$\mathbf{N}_B = c\mathbf{v}_B$

$$\mathbf{N}_A + \mathbf{N}_B = c\mathbf{v}_A + c\mathbf{v}_B \qquad (1)$$

$$\mathbf{V} = \frac{\sum c_i \mathbf{v}_i}{c} = \frac{c_A\mathbf{v}_A + c_B\mathbf{v}_B}{c} \qquad (24\text{-}14)$$

$$\Rightarrow \quad c\mathbf{V} = c_A\mathbf{v}_A + c_B\mathbf{v}_B \quad \blacksquare$$

b/ $\mathbf{n}_A + \mathbf{n}_B = \rho\mathbf{v}$

$\mathbf{n}_A$: massflöde relativt stationära koordinater

$\mathbf{v}$: massbaserad medelhastighet

$$\mathbf{n}_A + \mathbf{n}_B = \rho_A\mathbf{v}_A + \rho_B\mathbf{v}_B$$

$$\mathbf{v} = \frac{\sum \rho_i \mathbf{v}_i}{\rho} = \frac{\rho_A\mathbf{v}_A + \rho_B\mathbf{v}_B}{\rho} \qquad (24\text{-}13)$$

$$\Rightarrow \quad \mathbf{n}_A + \mathbf{n}_B = \rho\mathbf{v} \quad \blacksquare$$

c, $\mathbf{j}_A + \mathbf{j}_B = 0$

$\mathbf{j}$: massfluxet relativt massbaserad medelhastighet

Tabell 24.2 :

$$\left.\begin{aligned} \mathbf{j}_A &= -\rho D_{AB} \nabla \omega_A \\ \mathbf{j}_B &= -\rho D_{AB} \nabla \omega_B \end{aligned}\right\} \Rightarrow \mathbf{j}_A + \mathbf{j}_B = -\rho D_{AB} (\nabla \omega_A + \nabla \omega_B)$$

$$\omega_B = 1 - \omega_A \Rightarrow \nabla \omega_B = \nabla (1 - \omega_A) = -\nabla \omega_A$$

$$\Rightarrow \mathbf{j}_A + \mathbf{j}_B = -\rho D_{AB} (\nabla \omega_A - \nabla \omega_A) = 0 \quad \blacksquare$$

d, $\mathbf{J}_A + \mathbf{J}_B = 0$

$\mathbf{J}$: molfluxet relativt molbaserad medelhastighet

$$\left.\begin{aligned} \mathbf{J}_A &= -c D_{AB} \nabla y_A \\ \mathbf{J}_B &= -c D_{AB} \nabla y_B \end{aligned}\right\} \Rightarrow \mathbf{J}_A + \mathbf{J}_B = -c D_{AB} (\nabla y_A + \nabla y_B)$$

$\nabla y_B = \nabla (1 - y_A) = -\nabla y_A$ på samma sätt som i c,

$$\Rightarrow \mathbf{J}_A + \mathbf{J}_B = -c D_{AB} (\nabla y_A - \nabla y_A) = 0 \quad \boxtimes$$

17.3

		M (g/mol)	V (m/s)
CO	3%	28.01	5.1
CO_2	7%	44.01	3
O_2	11%	32	4.6
N_2	79%	28.014	5.1

$T = 21°C \qquad P = 1 \text{ atm}$

$\rho = \Sigma \rho_i \quad : \quad \rho_A = C_A M_A = y_A C M_A = y_A \frac{P}{RT} M_A$

$$v = \frac{\Sigma \rho_i v_i}{\rho} = \sum \frac{y_i \frac{P}{RT} M_i v_i}{y_i \frac{P}{RT} M_i} = 4.821714686 \text{ m/s}$$

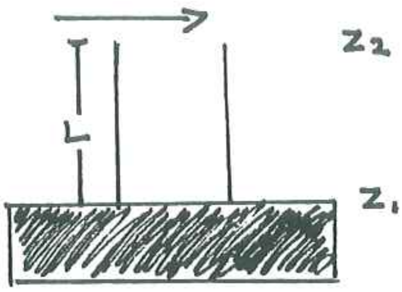
Pitotrör : $P_0 = P_1 + \frac{\rho v^2}{2}$

$\rho = 1.22523478 5 \text{ kg/m}^3$

$\Delta P = 14.2427004 1 = \rho g h$

$\Rightarrow h = 1.453 E^{-3} \text{ m} = \underline{\underline{0.15 \text{ cm}}}$

17.4}



Arnold cell: Diffusion genom
stagnant gasfilm

$A_{tv} = 0.82 \text{ cm}^2$

$T = 297 \text{ K}$

$P = 1.013E^5 \text{ Pa}$

$\Delta z = 15 \text{ cm}$

a, $\nabla N_A + \cancelto{0}{\frac{dC_A}{dt}} - \cancelto{0}{R_A} = 0$ steady state ≗ ingen reaktion

$\nabla N_A = 0$

$N_A = -cD_{AB} \nabla y_A + y_A (N_A + N_B)$

Stagnant gasfilm ⇒ $N_{Bz} = 0$

ämne B olösligt i A ⇒ $N_B = 0$ ≗ $\nabla N_B = 0$

$$N_{Az} = -\frac{cD_{AB}}{1-y_A} \cdot \frac{dy_A}{dz}$$

$$\int_{z_1}^{z_2} N_{Az}\, dz = -\int_{y_{A1}}^{y_{A2}} \frac{cD_{AB}}{1-y_A}\, dy_A$$

$\frac{dN_{Az}}{dz} = 0$ ⇒ N_{Az} konstant. Antag även cD_{AB}
oberoende av z ≗ y_A

$$N_{Az} \int dz = -cD_{AB} \int \frac{dy_A}{1-y_A}$$

$$\Rightarrow N_{Az} = \frac{cD_{AB}}{z_2-z_1} \ln\left(\frac{1-y_{A2}}{1-y_{A1}}\right) \qquad (*)$$

$$N_A \cdot A_{tv} = \frac{dn}{dt} \qquad n = \frac{V\rho}{M} \qquad t = 10\,h \qquad \Delta V = 0,0445\,cm^3$$

$$\Rightarrow N_A = \frac{\rho_{etanol}}{M_{etanol} \cdot A} \frac{\Delta V}{t}$$

$y_{A2} = 0$ oändigt utspädd

$c = P/RT$

$y_{A1} = \frac{P^{\circ}_{etanol}}{P}$

$$D_{AB} = \frac{\frac{\rho_e}{M_e} \Delta V \cdot \Delta z}{t A_{tv} \frac{P}{RT} \ln\left(\frac{1}{1-y_{A1}}\right)} \qquad \text{från } (*)$$

$$= \underline{\underline{13.4\,E^{-6}\ m^2/s}}$$

b, Hirschfelder:

$$D_{AB} = \frac{0.001858\,T^{3/2}\,(M_A^{-1} + M_B^{-1})^{1/2}}{P\,\sigma_{AB}^2\,\Omega_D}$$

A: etanol B: luft

Appendix K:

$\frac{\varepsilon_B}{K} = 97\text{ K}$ $\frac{\varepsilon_A}{K} = 391\text{ K}$

$\sigma_B = 3.617\text{ Å}$ $\sigma_A = 4.455\text{ Å}$

$$\sigma_{AB} = (\sigma_A + \sigma_B)/2 = 4.036\text{ Å}$$

$$\varepsilon_{AB} = (\varepsilon_A \varepsilon_B)^{1/2} = 194.7485558$$

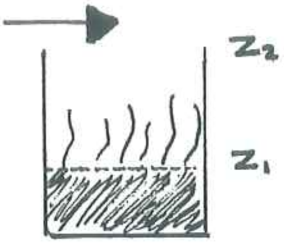
$$\frac{KT}{\varepsilon_{AB}} = 1.525 \Rightarrow \Omega_D = 1.19$$

$M_A = 46$ g/mol

$M_B = 29$ g/mol

$$\Rightarrow D_{AB} = \underline{\underline{11.6E^{-6}\ m^2/s}}$$

17.5 } Diffusion genom stagnant medium



$$N_{az} = \frac{D_{AB} \cdot P}{RT \Delta z} \cdot \frac{P_{A1} - P_{A2}}{P_{Blm}}$$

a, $y_A = \frac{P_A}{P}$ ⇒ om molbråken är konstanta (ändrar ej sammansättningen) så måste P_A fördubblas

⇒ $P_{A1} - P_{A2}$ fördubblas

samma gäller för P_B

$$P_{Blm} = \frac{P_{B2} - P_{B1}}{\ln\left(P_{B2}/P_{B1}\right)}$$ ⇒ P_{Blm} fördubblas

$\frac{P_{A1} - P_{A2}}{P_{Blm}}$ är konstant eftersom nämnare o täljare fördubblas

$D_{AB} \propto \frac{1}{P}$ (enligt tex Hirshfelder)

⇒ $D_{AB} \cdot P$ är konstant

∴ Fluxet ändras ej

b, $\frac{dP_B}{dz} \neq 0$ Hur kan då $N_{Bz} = 0$?

$$N_{Bz} = -cD_{AB}\frac{dy_B}{dz} + y_B(N_{Az} + N_{Bz})$$

$$= \underbrace{-\frac{c}{P}D_{AB}\frac{dP_B}{dz}}_{(1)} + \underbrace{\frac{P_B}{P}(N_{Az} + N_{Bz})}_{(2)}$$

(1) molårt flöde på grund av koncentrationsgradient

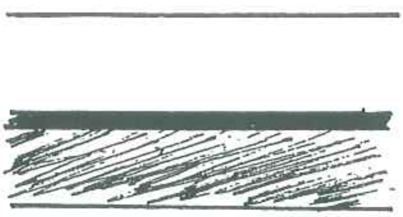
(2) molårt flöde på grund av bulktransport

∴ Diffusion & bulktransport motverkar varandra

När ämne A transporteras, drar den med sig molekyler av ämne B. Detta ger upphov till en koncentrationsgradient som medför att B diffunderar i motsatt riktning

Dessa två mekanismer balanserar varandra

17.6}



$Z_0 = 0$ cm $\quad Y_{NH_3} = 0{,}0342$

$z_1 = 2$ cm $\quad$ gränsyta

$z_2 = 3$ cm $\quad C_{NH_3} = 0$

$T = 15°C \qquad D_{ammoniak-luft} = 0.215\ cm^2/s$

$P = 1\ atm \qquad D_{ammoniak-vatten} = 1.77\ E^{-5} cm/s$

Transporten ammoniak genom gasfilmen är samma som transporten av ammoniak genom vätskefilmen

$\Rightarrow N_{NH_3 . gas} = N_{NH_3 . vätska}$

Gasfilmen: $N_{Ag} = \frac{cD_{AL}}{\Delta Z} \ln\left(\frac{1-Y_{A1}}{1-Y_{A0}}\right) \quad (26-5)$

$c = \frac{P}{RT} \Rightarrow N_{Ag} = \frac{PD_{AL}}{RT\Delta Z} \ln\left(\frac{1-Y_{A1}}{1-Y_{A0}}\right) \quad (1)$

Vätskefilmen: små mängder A löser sig i vatten ⇒ Fick's lag

$N_A = -D_{AV} \frac{dC_A}{dz} + Y_A(N_{Az} + N_{Bz}) \quad Y_A$ liten ⇒

$N_A = -D_{AV} \frac{dC_A}{dz}$

integrering ⇒ $N_{Az} = \frac{D_{AV}}{\Delta Z}(C_{A1} - C_{A2}) \quad (26-21) \quad (2)$

(1) = (2) $C_{A2} = 0$ i vatten

$$\frac{D_{AV}}{\Delta Z}(C_{A1} - C_{A2}) = \frac{P D_{AL}}{RT \Delta Z} \ln\left(\frac{1 - Y_{A1}}{1 - Y_{A0}}\right)$$

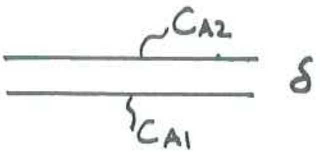
$$\Rightarrow C_{A1} = 256875.7797 \ln\left(\frac{1 - Y_{A1}}{0.9658}\right) \quad (*)$$

Rita diagram. Plotta P_{NH_3} mot C_{NH_3}

Gissa $P_{A1} \Rightarrow Y_{A1} \Rightarrow C_{A1}$ "diagram"
kontrollera C_{A1} med (*)

$$N_{AZ} = \frac{D_{AV}}{\Delta Z}(C_{A1} - C_{A2}) = \underline{\underline{11.9\,E^{-6}}}\ \mathrm{mol}/\mathrm{m}^2\mathrm{s}$$

17.7}



$P = 101.3\,E^{3}$ Pa

$A = 1\ m^2$

$C_{A1} = 0{,}04\ mol/m^3$

$C_{A2} = 0{,}01\ mol/m^3$

a, luft 289 K

$$N_{Az} = \frac{c D_{AB}}{\Delta z} \ln\left(\frac{1-y_2}{1-y_1}\right)$$

Appendix J: $P D_{AB}\ (273K) = 1.256\ m^2\,Pa/s$

$$\left.\begin{array}{l} \frac{\varepsilon_{Cl}}{k} = 357 \\ \frac{\varepsilon_{luft}}{k} = 97 \end{array}\right\} \quad \frac{\varepsilon}{k} = \frac{186.089}{k}$$

$$D_{ABT_2} = \frac{P_1 D_{AB|T_1}}{P}\left(\frac{T_2}{T_1}\right)^{1.5} \frac{\Omega_{D|T_1}}{\Omega_{D|T_2}}$$

$$\Omega_{D|T_1} = f\left(\frac{Tk}{\varepsilon}\right) = f(1.467) = 1.209$$

$$\Omega_{D|T_2} = f\left(\frac{Tk}{\varepsilon}\right) = f(1.553) = 1.182$$

$$\Rightarrow D_{ABT_2} = 1.3813\,E^{-5}\ m^2/s$$

$c = P/_{RT} = 42.16\ mol/_{m^3}$

$Y_1 = \frac{C_{A1}}{C} = 0.00095$

$Y_2 = \frac{C_{A2}}{C} = 0.00024$

$\Rightarrow \quad N_{Az} = 8.293\ E^{-5}\ mol/m^2 s$

$N_{Az} = \frac{n}{At}$ enheter! $\Rightarrow \quad t = 12058.8 s = \underline{\underline{3.35\ h}}$

b) vatten 289 K

$D_{AB} = 1.26\ E^{-9}\ m^2/s$

$C_T = 55505.6\ mol/_{m^3}$

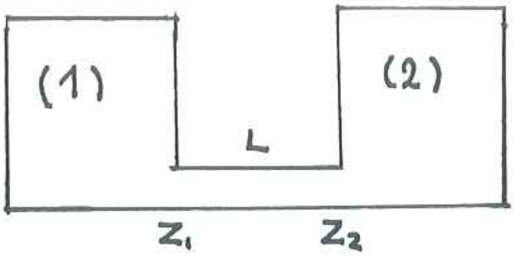
$y_1 = 7.21\ E^{-7}$

$y_2 = 1.8\ E^{-7}$

$N_{Az} = 7.56\ E^{-9}\ mol/_{m^2 s}$

$\Rightarrow \quad t = 1.32\ E^{8}\ s = \underline{\underline{4.2\ \text{år}}}$

17.8



A = ammoniak

B = luft

L = 3m

D = 0,15 m

T = 273 K

P = 1.013 E^5 Pa

y_{B1} = 0.4

y_{B2} = 0.8

y_{A1} = 0.6

y_{A2} = 0.2

Fick's lag: $N_{Az} = -cD_{AB}\frac{dy_A}{dz} + y_A(N_{Az} + N_{Bz})$

Ekvimolekylär motdiffusion. Eftersom P = V är konstanta i de bägge tankarna så strömmar lika många mol från (1) till (2) som från (2) till (1)

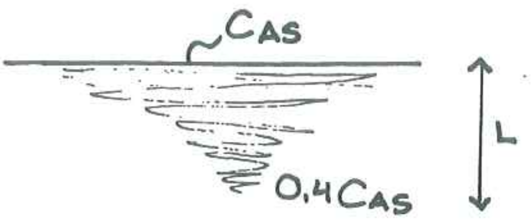
$$\int N_A dz = -\int cD_{AB} dy_A \Rightarrow N_{Az} = -cD_{AB}\frac{(y_{A2} - y_{A1})}{L}$$

Appendix J: $D_{AB} \cdot P = 2.006$ Pa m^2/s

$\Rightarrow N_{Az} = 1.18 E^{-4}$ mol/m^{2}s

$$N_{Az} \cdot A = N_{Az} \cdot \frac{\pi D^2}{4} = \underline{\underline{2.08 E^{-6} \text{ mol/s}}} = -N_{Bz} A$$

18.1



$D_{AB} = 1E^{-7}\ cm^2/s$

$C_{A(L)} = 0.4\ C_{A0}$

Diffusion genom semioändlig platta

C_{AS} = konstant ↔ $m = 0$ ⇒

$$\frac{C_A - C_{A0}}{C_{AS} - C_{A0}} = 1 - \mathrm{erf}\left(\frac{z}{2\sqrt{D_{AB}t}}\right)$$

där $C_A = 0.4 C_{A0}$

$\underline{o}$ $C_{AS} = 0$

a, $t = 3600\ s \Rightarrow \frac{L}{0.0379} = 0.37 \Rightarrow L = \underline{\underline{0.14\ mm}}$

b, $t = 36000\ s \Rightarrow L = \underline{\underline{0.44\ mm}}$

18.2 }

$D = 13\,mm$ $\qquad C_A(0t_1) = 0.27\,C_{A0}$

$t_1 = 30\,h$ $\qquad C_{ut} = C_A(0t_2)$

$Y = \dfrac{C_{AS} - C_A}{C_{AS} - C_{A0}}$ Väl omrörd $\Rightarrow C_{AS} = 0 \Rightarrow$

$Y = \dfrac{C_A}{C_{A0}}$

$n = m = 0$

$X_D = \dfrac{D_{AB}\,t}{x_1^2}$

$Y_1 = \dfrac{C_A(0t_1)}{C_{A0}} = 0.27 \Rightarrow X = 0.18 \Rightarrow D_{AB} = 1.667\,E^{-6}$

$t_2 = 37.4\,h \Rightarrow X = 0.2244 \Rightarrow Y = \underline{\underline{0.15}} = \dfrac{C_A(0t_2)}{C_{A0}}$

∴ $\underline{\underline{\text{15\% av ursprungsvärdet}}}$

18.3}

$\delta = 5 cm$ $\omega_{A0} = 0.15$

$D_{AB} = 1.3E^{-4} cm^2/s$ $\omega_{AS} = 0.04$

$\omega_A = 0.1$

Ändeffekter försummas

$$\omega_A = \frac{C_A \cdot M_A}{C_A M_A + C_B M_B} \Rightarrow C_A = \frac{\omega_A C_B M_B}{M_A(1-\omega_A)}$$

$$Y = \frac{C_{AS} - C_A}{C_{AS} - C_{A0}} = 0.515152$$

Är bara ovansidan som transporterar fukt ⇒
x_1 blir dubbelt så tjock $x_1 = \delta$

$n = 0.5$
$m \approx 0$ inget konvektivt motstånd

$n = 0.5$ finns ej. Ta medelvärdet för $n = 0.4$ & $n = 0.6$

$$\Rightarrow X = 0.22 = \frac{D_{AB} t}{x_1^2} = 5.2 E^{-6} t$$

$$\Rightarrow t = 42307.75 \approx 11.75 h \approx \underline{\underline{12 h}}$$

18.4} $\rho_{A0} = 2 kg/m^3$ $\rho_{AS} = 9 kg/m^3$ $T = 283 K$

Semi infinit medium

$$\frac{C_A - C_{A0}}{C_{AS} - C_{A0}} = 1 - erf\left(\frac{z}{2\sqrt{D_{AB} \cdot t}}\right) \qquad (*)$$

$$C_A = \frac{\rho_A}{M_A} \Rightarrow$$

$$\rho_A = -7 \cdot erf\left(\frac{z}{2\sqrt{D_{AB} \cdot t}}\right) + 9 \qquad \text{från } (*)$$

D_{AB} fås från Wilke & Chang

$$\frac{D_{AB}\,\mu_B}{T} = \frac{7.4E^{-8}(\phi_B M_B)^{1/2}}{V_A^{0.6}}$$

$\mu_B = \mu_{H_2O} = 1393.5\,E^{-6}$ Pas $= 1.3935$ cP

$M_B = M_{H_2O} = 18$ g/mol

$\phi_B = 2.26$ (sid 496)

$V_A = 25.6\ cm^3/gmol$ (sid 495)

$$\Rightarrow D_{AB} = 13.69801619\,E^{-6}\ cm^2/s$$

a, $t = 3600$ s $\rho_A = 9 - 7\,\mathrm{erf}\left(135.0955883 \cdot \frac{z}{\sqrt{t}}\right)$

z (cm)	ϕ	erf (ϕ)	ρ_A
0	0	0	9.0
0.011	0.02477	0.0282	8.8026
0.022	0.0495	0.0564	8.6052
0.044	0.0991	0.1125	8.2125
0.089	0.2004	0.2227	7.4411
0.178	0.4008	0.4284	6.0012
0.355	0.7993	0.7421	3.8053

plotta ρ_A mot z

precis likadant för uppgift b: c, dock med andra tider

18.5} $\omega_{A0} = 0.45$ $\omega_A = 0.25$ $\delta = 0.05\,m$

$\omega_{AS} = 0.14$ $D_{AB} = E^{-9}\ m^2/s$

$C_A = \dfrac{\omega_A C_B M_B}{(1-\omega_A)\, M_A}$ för detaljer, se uppgift 16.3

Diagramlösning

$Y = 0.26$

a, $m = 0.25$ $n = 0$

$\Rightarrow X = 0.98 \Rightarrow t = 612500\ s = \underline{\underline{170\ h}}$

b, $m = 0$ $n = 0$

$\Rightarrow X = 0.65 \Rightarrow t = 406250\ s = \underline{\underline{113\ h}}$

18.6}

$\omega_{AO} = 0.45$

$\omega_{AS} = 0.05$

$\rho_A = 800\ kg/m^3$

$L = 15\ m$

$\phi = 2.5E^{-3}m$

$T = 47°C$

$D_{AB} = 2E^{-8}\ m^2/s$

$D = 0.8\ m$

$\omega_{A(r=0)} = 0.25$

$C_B = C_{BS} = C_{BO} \Rightarrow$

$$Y = \frac{\frac{\omega_{AS}}{1-\omega_{AS}} - \frac{\omega_A}{1-\omega_A}}{\frac{\omega_{AS}}{1-\omega_{AS}} - \frac{\omega_{AO}}{1-\omega_{AO}}} = 0.3667$$

$n = m = 0$ försummar yttre motstånd

$\Rightarrow X = 0.16 \Rightarrow t = 12.5\ s$

Kraftbalans på en fallande kropp

$F_{formmotstånd} = F_{tyngd}$ $\qquad F_{lyft} \approx 0$

$$F_{tyngd} = mg = \rho \cdot \frac{4\pi\phi^3}{24} \cdot g$$

$$F_f = C_D \cdot \rho \frac{Av^2}{2}$$

$$V = \frac{Q}{A} + V_{partikel}$$

$$C_D = f(Re) = f\left(\frac{v\varnothing}{\nu}\right)$$

$$V_{partikel} = \frac{L}{t} = 1.2\ m/s$$

Gissa $C_D \Rightarrow v \Rightarrow Re \Rightarrow C_D$

$$C_D = 0.5$$

$$Q = (v - V_{partikel})\frac{\pi D^2}{4} = 2.88 \approx \underline{\underline{2.9}}\ m^3/s$$

$\sum R = \frac{\ln(r_B/r_A)}{2\pi k L}$ för ledning

$R_1 = \frac{\ln(r_2/r_1)}{2\pi k_s L} \Rightarrow LR_1 = 3.611E^{-3} \frac{mK}{W}$ (1)

$R_2 = \frac{\ln(r_3/r_2)}{2\pi k_1 L} \Rightarrow LR_2 = 0.122553 \cdot k_1^{-1}$ (2)

$R_3 = \frac{\ln(r_4/r_3)}{2\pi k_2 L} \Rightarrow LR_3 = 0.0684097 \cdot k_2^{-1}$ (3)

$L \sum R = (1) + (2) + (3)$

Glasull närmast: $k_1 = 0.038$ W/mK $k_2 = 0.066$ W/mK

$L \sum R = 4.2652 \, (W/mK)^{-1}$

Magnesium närmast: $k_1 = 0.066$ W/mK $k_2 = 0.038$ W/mK

$L \sum R = 3.66073$ mK/W

a) Glasull närmast isolerar bäst

19.1}

$$Nu = 0.332\ Re_x^{1/2}\ Sc^{1/3}$$ laminärt

$$Nu = 0{,}0292\ Re_x^{4/5}\ Sc^{1/3}$$ turbulent

a, $Re_L = 100\,000$ omslag vid $Re = 3E^5$

Sätt plattans längd till $L \Rightarrow \frac{v\rho}{\mu} = \frac{100\,000}{L}$

Omslag vid $L_{kr} \Rightarrow Re = \frac{vL_{kr}\rho}{\mu} = \frac{L_{kr}\cdot 100\,000}{L}$

$Re = 3E^5 \Rightarrow L_{kr} = 3L$

∴ laminärt på hela plattan

$$\bar{k}_{cL} = \frac{1}{L}\int k_{cx}\,dx = \frac{D_{AB}}{L}\int \frac{Nu}{x}\,dx$$

$$= \frac{1}{L}\int \frac{D_{AB}\cdot 0.332\cdot Re_x^{1/2}\cdot Sc^{1/3}}{x}\,dx$$

$$= \frac{2\cdot 0.332}{L}\cdot Sc^{1/3}\cdot Re_L\, D_{AB}$$

$$= \frac{210\ Sc^{1/3}}{L}D_{AB}$$

b, $Re_L = 1.5E^6 \Rightarrow L_{kr} = 0.2L$

$$\bar{k}_c = \frac{\int_0^{L_{kr}} k_c\,dx \text{ (laminärt)} + \int_{L_{kr}}^{L} k_c\,dx \text{ (turbulent)}}{L}$$

uttryck för k_c i det laminära fallet fås från a,

$$\int_{L_{kr}}^{L} k_c\,dx = \int \frac{0.0292\, Re_x^{4/5} \cdot Sc^{1/3}\, D_{AB}}{x}\,dx$$

$$\bar{k}_c = \frac{Sc^{1/3} D_{AB}}{L} \cdot \left[0.664\, Re_{L_{kr}}^{1/2} + 0.0365\left(Re_L^{4/5} - Re_{L_{kr}}^{4/5}\right)\right]$$

$$= 2670 \cdot \frac{D_{AB}\, Sc^{1/3}}{L}$$

19.2} $\dot{m} = 0.045$ kg/h då $v = 6.1$ m/s

Vet ej T_{luft}. Antag 280K ⇒ $Re = 1.3E^5$ laminärt !

$$k_c = \frac{D_{AB}}{x}\left[0.332\, Re_x^{1/2}\right]$$

$$k_{cL} = \frac{1}{L} D_{AB}\, 0{,}332 \int_0^L \frac{Re_x^{1/2}}{x}\, dx = 0.664 \frac{D_{AB}\, Re_L^{1/2}}{L}$$

$$N_A = k_{cL}\left(C_{As} - C_{A\infty}\right)$$

$C_{As} = 0.0458$ mol/m^3

$C_{A0} = 0$ ny luft kommer in hela tiden

$$\dot{m} = N_A \cdot M_{H_2O} \cdot A \Rightarrow k_{cL} = \frac{0.6068}{3600}$$

$$\Rightarrow \frac{D_{AB} \cdot Re_L^{1/2}}{L} = \frac{0.914}{3600}$$

Hastigheten halveras från 6.1 till 3.05 m/s

D_{AB} är konstant   L är konstant

$Re_{L}\,ny = \frac{1}{2} Re_{L}$

$$\Rightarrow \frac{D_{AB}\cdot Re_{L\,ny}^{1/2}}{L} = \frac{D_{AB}\cdot Re_{L}^{1/2}}{L\cdot\sqrt{2}} = \frac{0.6462}{3600}$$

$$\Rightarrow k_{cL} = 0.664\cdot\frac{D_{AB}\,Re_{L\,ny}^{1/2}}{L} = \frac{0.429}{3600}$$

$$N_{A\,ny} = k_{cL}\,(C_{AS}-C_{A\infty}) = 0.0196\ mol/m^2s$$

$$\Rightarrow \dot{m} = 32\ g/h$$

19.3)

$$Nu_D = 0.37\, Re_D^{0.6} \cdot Pr^{1/3} \qquad 20 < Re_D < 150\,000$$

Chilton - Colburn behövs för att få fram ett uttryck för massöverföringen

$$j_D = j_H \qquad j_D = St_D \cdot Sc^{2/3}$$

$$j_H = St_H \cdot Pr^{2/3}$$

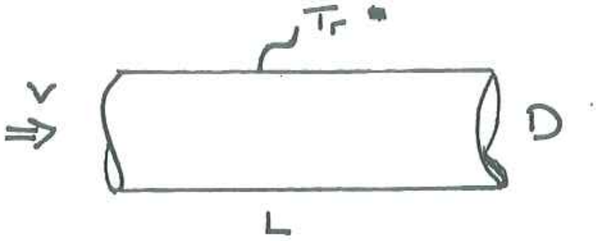
$$St_D = \frac{Sh}{Re \cdot Sc} \qquad St_H = \frac{Nu}{Re\,Pr}$$

$$j_D = \frac{Sh}{Re\, Sc^{1/3}}$$

$$j_H = \frac{Nu}{Re\, Pr^{1/3}} = \frac{0.37}{Re^{0.4}}$$

$$\Rightarrow \quad Sh = 0.37 \cdot Re^{0.6} \cdot Sc^{1/3}$$

19.4 }



$T_r = 290K$

$L = 6m$

$P = 1 atm$

$T_i = 310 K$

$V = 1.5 m/s$

$D/e = 10'000$

$P_v^\circ = 19.37$ mbar

$D = 0.15 m$

1, Chilton-Colburn

$$j_D = \frac{C_f}{2}$$

$$\left.\begin{array}{l} Re = 13530 \\ e/D = E^{-5} \end{array}\right\} C_f = 0.007 \quad \text{figur 14-1}$$

$$\Rightarrow \quad j_D = 3.5\,E^{-3}$$

2, $j_D = St_D \cdot Sc^{2/3} = \frac{k_c}{V} Sc^{2/3}$

$Sc = \frac{\nu}{D_{AB}}$ $\quad$ $P \cdot D_{AB} = 2.634$

$$D_{ABT_2} = D_{ABT_1}\left(\frac{T_2}{T_1}\right)^{3/2}\frac{\Omega_D|_{T_1}}{\Omega_D|_{T_2}}$$

$$\Omega_D = f\left(\frac{KT}{\varepsilon}\right)$$

$$\left.\begin{array}{ll} K/\varepsilon_{\,luft} & = 97 \\ K/\varepsilon_{\,vatten} & = 356 \end{array}\right\} \quad \frac{\varepsilon}{K} = \frac{\sqrt{\varepsilon_l \cdot \varepsilon_v}}{K} = 185,83$$

$$\frac{298}{185.8} = 1.6 \Rightarrow \Omega_D = 1.167$$

$$\frac{310}{185.8} = 1.67 \Rightarrow \Omega_D = 1.151$$

$$\Rightarrow P D_{ABT_2} = 2.8335\ m^2 Pa/s$$

$$\Rightarrow Sc = 0.59346$$

$$\Rightarrow k_c = 7,434\ \varepsilon^{-3}\ m/s$$

$N_A = k_c \Delta C$ lokalt

$$\Delta C_{lm} = \frac{\Delta C_{in} - \Delta C_{ut}}{\ln\left(\frac{\Delta C_{in}}{\Delta C_{ut}}\right)}$$

$N_{ATOT} = k_c \Delta C_{lm}$ känner ej $C_{A\infty}$ ut

$C_{A\infty} = 0$ torr luft

$C_{ASin} = C_{ASut} = \frac{P_v^\circ}{RT} = 7{,}515 E^{-1}$ mol/m^3

$$\left.\begin{array}{l}\text{vatten in} = N_{TOT} \cdot A_{mantel} \\ \text{vatten ut} = C_{A\infty ut}\, A_{tv} \cdot v\end{array}\right\} \text{lika med varandra}$$

Gissa $C_{A\infty out} \Rightarrow N_{TOT} \Rightarrow$ vatten in $\Rightarrow C_{A\infty}$ ut

$\Rightarrow C_{A\infty out} = 4.1174 E^{-1}$ mol/m^3

3, $N_{TOT} = 3{,}86 E^{-3}$ från iterationen

vatten in $= \dot{m}_{H_2O\text{ tillsatt}} = 1.96 E^{-1}$ g/s

Svar: Man måste tillsätta 0,2 g/s

19.5}

$t_\infty = 38^\circ c$ $\quad V = 1.5\ m/s$

$D = 1.9\ cm$ $\quad D_{AB} = 6.51\ E^{-6}\ m^2/s$

$\nu = 1.68\ E^{-5}\ m^2/s$ $\quad \alpha = 2.37\ E^{-5}\ m^2/s$

$\rho = 1.14\ kg/m^3$ $\quad k = 0.027\ W/m$

$P = 1\ atm$ $\quad P_A^\circ = 5\ mm\ Hg$

a, Påtvingad konvektion:

$$Nu_D = 2 + 0.6\, Re_D^{1/2}\, Pr^{1/3} = 24.03 \Rightarrow h = \underline{\underline{34.2\ W/m^2K}}$$

b, Chilton-Colburn: $j_H = j_D$

$$\frac{h}{\rho c_p} \cdot Pr^{2/3} = k_c\, Sc^{2/3} \Rightarrow k_c = \underline{\underline{0.0127\ m/s}}$$

c, $$N_A = k_c \Delta C = \frac{k_c}{RT}\left(P_A^\circ - P_{A\infty}^\circ\right)$$

$$P_{A\infty}^\circ = 0 \Rightarrow N_A = \underline{\underline{3.27\ E^{-3}\ mol/m^2 s}}$$

19.6}

$L = 3 m$ $\quad$ $D_{AB} = 9.51 E^{-6}\ m^2/s$

$B = 1.5 m$ $\quad$ $P_A^\circ = 8\ hPa$

$T_A = 289 K$ $\quad$ $v_l = 5 m/s$

$T_\infty = 303 K$ $\quad$ $P = 1.013 E^5\ Pa$

$$Re = \frac{v \cdot B}{\nu} = 4.892\ E^5 \qquad T_f = 296 K$$

plattan är delvis turbulent

$$\bar{k}_c = \frac{Sc^{1/3} D_{AB}}{L} \cdot \left[0.664\, Re_{Lkr}^{1/2} + 0.0365 \left(Re_L^{4/5} - Re_{Lkr}^{4/5} \right) \right]$$

för detaljer, se uppgift 17.1

$L = B \qquad Re_{Lkr} = 2 E^5$

$$\Rightarrow \bar{k}_c = 7.146\ E^{-3}\ m/s$$

$$N_A = \bar{k}_c \left(c_A - c_{A\infty} \right)$$

$$c_{A\infty} = 0 \qquad c_A = \frac{P_A^\circ}{R T_A}$$

$$\Rightarrow N_A = 23.79\ E^{-3}\ mol/m^2 s$$

$$\dot{m} = N_A \cdot M_A \cdot A = \underline{\underline{8.35\ g/s}}$$

b, $v = 3\ cm/s \Rightarrow Re = 2{,}93 E^{10}$

hela plattan är laminär

$$\overline{k_c} = \frac{Sc^{1/3} D_{AB}}{B} \cdot 0.664\ Re^{1/2} = 2{,}67 E^{-4}\ m/s$$

$$N_A = \overline{k_c} \cdot C_A = \underline{\underline{8{,}9\ mol}}/m^2s$$

19.7} $Nu_{AB} = 2 + C Re^{m} Sc^{1/3}$

$$\nabla N_A + \frac{dC_A}{dt} - R_A = 0$$

1, Stationärt $\Rightarrow \frac{dC_A}{dt} = 0$

2, ingen reaktion $\Rightarrow R_A = 0$

$\Rightarrow \nabla N_A = 0$ uttryckes i sfäriska koordinater

3, endast transport i radiell led

$\Rightarrow \frac{d}{dr}\left(r^2 N_{Ar}\right) = 0$ $\qquad N_{Ar} = N_A$

integreras $\Rightarrow r^2 N_A = C_1 \Rightarrow N_A = \frac{C_1}{r^2}$

Fick's lag: $N_A = -D_{AB}\frac{dC_A}{dr} \Rightarrow \frac{C_1}{r^2} = -D_{AB}\frac{dC_A}{dr}$

integreras $\Rightarrow C_A = \frac{C_1}{rD_{AB}} + C_2$

$$\left.\begin{matrix} r = R \Rightarrow C_A = C_{Ay} \\ r = \infty \Rightarrow C_A = C_{A\infty} \end{matrix}\right\} C_A = \left(C_{Ay} - C_{A\infty}\right)\frac{R}{r} + C_{A\infty}$$

$$N_A = \frac{C_1}{r^2} = \left(C_{Ay} - C_{A\infty}\right)\frac{R}{r}D_{AB}$$

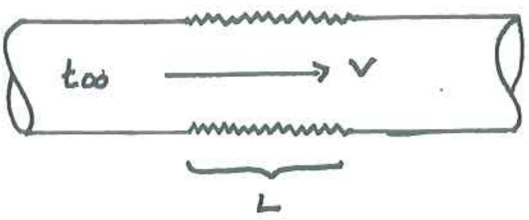
Masstransport vid ytan kan beskrivas med hjälp av k_c

$$\left.\begin{array}{l} N_A = k_c\,(C_{Ay} - C_{A\infty}) \\ N_A = N_{A(r=R)} = \frac{R}{R^2} D_{AB}\,(C_{Ay} - C_{A\infty}) \end{array}\right\} \quad k_c = \frac{D_{AB}}{R}$$

$$Nu_{AB} = \frac{k_c D}{D_{AB}} = \frac{D_{AB} \cdot D}{D_{AB} \cdot R} = \frac{D}{R}$$

$$D = 2R \quad \Rightarrow \quad Nu_{AB} = 2 \qquad ▨$$

19.8



$D = 0.051$ m

$L = 0.65$ m

$t_\infty = 21°C$

$v = 3$ m/s

$\nu = 0{,}995\,E^{-6}$

$\left(D_{AB} = 1{,}26\,E^{-9}\ \frac{m^2}{s} \right.$
Appendix J $\left. \right)$

a, $Nu_{AB} = \frac{k_L D}{D_{AB}} = 0.023\ Re^{0{,}83}\ Sc^{1/3}$

$\left. \begin{array}{l} Re = 1.538\,E^{5} \\ Sc = 790 \end{array} \right\}\ k_L = \underline{\underline{1.06\ E^{-4}}}$ m/s

b, Chilton-Colburn : $j_D = \frac{C_f}{2}$

$\frac{C_f}{2} = \frac{k_L}{V_\infty} Sc^{2/3}$

$Re = 1.538\,E^{5} \Rightarrow C_f = 0{,}0042$

$\Rightarrow\ k_L = \underline{\underline{0.737\ E^{-4}}}$ m/s

20.1}

$P_{tot} = 105\,E^3\ Pa$ $\qquad T = 310K$

$P_v^\circ(300K) = 3.6\ kPa$ $\qquad T_v = 300K$

$P_v^\circ(310K) = 6.33\ kPa$ $\qquad \lambda = 2440\ kg/kg$

$$q A^{-1} = h\Delta T = \lambda M_v \cdot N_v$$

$$N_v = k_c \Delta C$$

Chilton-Colburn: $j_H = j_D \Rightarrow \frac{k_c}{h} = \frac{1}{\rho C_p}\left(\frac{Pr}{Sc}\right)^{2/3}$

$$\frac{\Delta T}{\lambda M_v} = \frac{N_v}{h} = \frac{k_c \Delta C}{h}$$

Data tas vid $T_f = 305K$

$$C_{vs} = \frac{P_{vAs}}{RT} = \frac{P_{vs}^\circ}{RT}$$

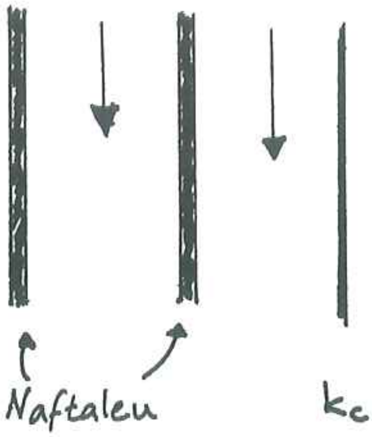
$$C_{v\infty} = C_{vs} - \frac{\Delta T}{\lambda M_v} \cdot \frac{h}{k_c} = 1.193\ mol/m^3$$

$$P_{v\infty} = C_{v\infty} \cdot R \cdot T_\infty = 3074.5\ Pa$$

Relativ fuktighet $\varphi = \frac{\text{fukthalt}}{\text{maximal fukthalt}} = \frac{P_{v\infty}}{P_{v\infty}^\circ}$

$$\varphi = \frac{3074}{6330} = 0.486$$

20.2}



$k_c = L/t \qquad \rho = M/L^3$

$v = L/t \qquad \mu = M/tL$

$D_o = L \qquad D_{AB} = L^2/t$

$D_i = L$

$k_c = k_c(v, D_o, D_i, \rho, \mu, D_{AB})$

$$\begin{array}{c|ccccccc} & k_c & v & D_o & D_i & \rho & \mu & D_{AB} \\ \hline L & 1 & 1 & 1 & 1 & -3 & -1 & 2 \\ t & -1 & -1 & 0 & 0 & 0 & -1 & -1 \\ M & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{array} \sim \begin{array}{ccccccc} 1 & 1 & 1 & 1 & -3 & -1 & 2 \\ 0 & 0 & 1 & 1 & -3 & -2 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{array}$$

$\Rightarrow$ rang 3 $\Rightarrow$ 3 kärnvariabler

ρ beskriver fluiden $\qquad$ D_o längd

D_{AB} beskriver diffusionen

(1) $\Pi_1 = D_o^a \, D_{AB}^b \, \rho^c \, k_c$

$L^a \left(\frac{L^2}{t}\right)^b \left(\frac{M}{L^3}\right)^c \cdot \frac{L}{t}$

$M: \; c = 0$

$t: \; -b - 1 = 0 \;\Rightarrow\; b = 1$

$L: \; a + 2b - 3c + 1 = 0 \;\Rightarrow\; a = 1$

$$\left.\begin{array}{l} \\ \\ \end{array}\right\} \Pi_1 = \frac{D_o}{D_{AB}} k_c$$

(2) $\Pi_2 = D_o^{d} D_{AB}^{e} \rho^{f} \cdot v$

$$L^{d}\left(\frac{L^2}{t}\right)^{e}\left(\frac{M}{L^3}\right)\frac{L}{t} \Rightarrow \Pi_2 = \frac{D_o}{D_{AB}} \cdot v$$

(3) $\Pi_3 = D_o^{g} D_{AB}^{h} \rho^{i} \cdot D_i$

$$L^{g}\left(\frac{L^2}{t}\right)^{h}\left(\frac{M}{L^3}\right)^{i} \cdot L \Rightarrow \Pi_3 = \frac{D_i}{D_o}$$

(4) $\Pi_4 = D_o^{j} D_{AB}^{k} \cdot \rho^{l} \cdot \mu$

$$L^{j}\left(\frac{L^2}{t}\right)^{k}\left(\frac{M}{L^3}\right)^{l} \cdot \frac{M}{tL} \Rightarrow \Pi_4 = \frac{\mu}{\rho D_{AB}}$$

20.3}

$D = 4 mm$ $v = 4 m/s$ Löslighet $= 1 kg/m^3$

$T = 15°C$ $P = 1 atm$ $D_{AB} = 1,77E^{-9} m^2/s$

Medelfluxet för en exponeringstid

$$\overline{N_A} = 2 \Delta C \sqrt{\frac{D_{AB}}{\pi t}}$$

$$t = \frac{L}{v} = 0.05 s$$

$$C_{A\infty} = 0$$

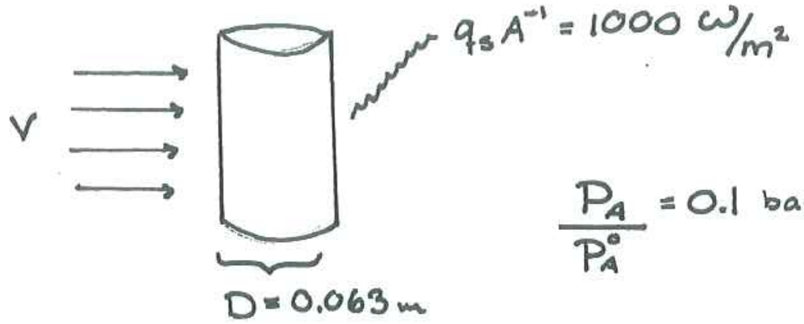
$$C_{AS} = \frac{1}{M_{NH_3}} = 58.74 mol/m^3$$

$$A = \pi D \cdot L$$

$$\dot{n} = \overline{N_A} \cdot A = 3,134 mol/s$$

$$\dot{m} = \dot{n} M = 5.33E^{4} g/s$$

20.4}



$T_\infty = 50°C$

$\frac{P_A}{P_A^\circ} = 0.1$ bar ⇒ $P_A = 1233.5$ Pa

Totalt tillförd värme = förångning av vatten

$A(h\Delta T + q_s A^{-1}) = \lambda \dot{m}$

$\dot{m} = N_v \cdot M_v \cdot A$

$N_v = k_c(C_s - C_\infty) = k_c\left(\frac{P_{AS}^\circ}{RT_s} - \frac{P_A}{RT_\infty}\right) = k_G(P_{AS}^\circ - P_A)$

Vinkelrät strömning mot cylinder

$Nu_D = B Re^n Pr^{1/3}$ B n beroende av Re

$T_f = \frac{T_s + T_\infty}{2}$

$\frac{k_G \cdot P \cdot Sc^{0,56}}{P \cdot D_{AB}} = 0.281 Re_D^{-0,4}$

Re_D för högsta värdet då $T = 320K = 59660$

$Re_D > 25000$ ⇒ vi får använda Chilton-Colburn

$$j_H = j_D \Rightarrow \frac{h}{\rho C_p} Pr^{2/3} = k_c Sc^{2/3}$$

$$Sc = \frac{\nu}{D_{AB}}$$

$$D_{ABT_2} = D_{ABT_1} \left(\frac{T_2}{T_1}\right)^{3/5} \frac{\Omega_{DT_1}}{\Omega_{DT_2}}$$

Gissa $T_s \Rightarrow T_f \Rightarrow Re\ B\ n\ D_{ABT_2} \Rightarrow Nu_D\ Sc$

$\Rightarrow h\ k_c \Rightarrow N_v \Rightarrow \dot{m}$

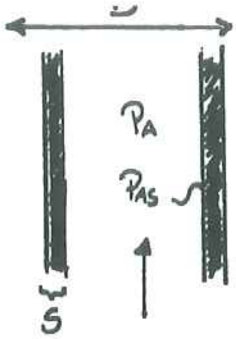
kontrollera om $A(h\Delta T + q_s A^{-1}) = \lambda \dot{m}$

Lättast är att gissa två temperaturer, en "låg" & en "hög" för att sedan säga att T_s ligger mellan dessa två

$$\underline{\underline{T_s = 25.5°C}}$$

Istället för att jämföra VL & HL kan den läggas in i miniräknarens "solverfunktion". Är dock en tråkig ekvation att skriva in eftersom det finns många ställen man kan göra fel.

21.1 }



$D = 3 cm$

$T = 298 K$

$P = 1.013 E^5 Pa$

$Q = 9.5 E^{-4} m^3/s$

$P_A = 665 Pa$

$\nu = 1.7 E^{-5} m^2/s$

$P_A^\circ = 3166 Pa$

$k_G = f(Re, Sc)$

$$Re_D = \left[\begin{array}{l} v \approx Q/A \\ s \text{ är liten} \Rightarrow A = \frac{\pi D^2}{4} \end{array} \right] = 2372$$

$$Sc = \frac{\nu}{D_{AB}} \qquad D_{AB} P = 2.634 \frac{m^2 Pa}{s}$$ (appendix J vatten i luft)

$\Rightarrow Sc = 0.654$

För turbulent strömning i rör med aktuell Sc gäller:

$$\frac{k_c D}{D_{AB}} \cdot \frac{P_{Blm}}{P} = 0.023 Re^{0.83} Sc^{0.44} \qquad (30-12)$$

$$P_{Blm} = \frac{\overset{\text{bulk}}{P_B} - \overset{\text{vid ytan}}{P_{BS}}}{\ln\left(\frac{P_B}{P_{BS}}\right)} \qquad \begin{array}{l} P_B = P - P_A \\ P_{BS} = P - P_A^\circ \end{array}$$

$\Rightarrow P_{Blm} = 9.958 E^4$

$$\Rightarrow \quad k_c = 0.01065 \text{ m/s}$$

$$N_A = k_G(P_A - P_{AS}) = k_c(C_A - C_{AS}) = k_c\left(\frac{P_A}{RT} - \frac{P_{AS}}{RT}\right)$$

$$\Rightarrow \quad k_G = \frac{k_c}{RT} = \underline{\underline{4.3E^{-6}}} \text{ mol}/\text{m}^2\text{sPa}$$

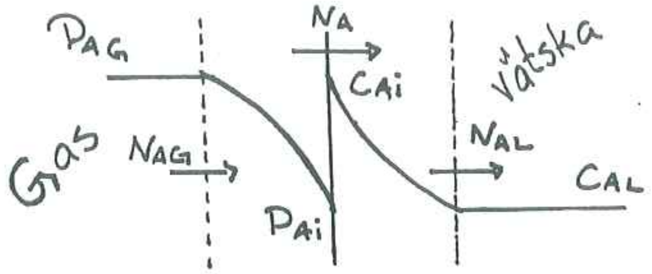
21.2 }

$k_c = 0.01065$

$$k_c = \frac{D_{AB} \cdot P}{\delta \, P_{Blm}} \qquad (26\text{-}9)$$

$$\left.\begin{aligned} D_{AB} \cdot P &= 2.634 \\ P_{Blm} &= 9.958\,E^4 \end{aligned}\right\} \Rightarrow \delta = 2.48 \text{ mm} \approx \underline{\underline{2.5 \text{ mm}}}$$

21.3



$T = 20°C$

$P = 1 \text{ atm}$

$P_{AG} = 30 \text{ mm Hg} = 4001 \text{ Pa}$

$C_{AL} = 0.55 \text{ kmol/m}^3$

$K_L = 1.1 E^{-4} \text{ m/s}$

$K_G = 3.95 E^{-6} \text{ mol/m}^2\text{s}$

P_A (mmHg)	0.55	3.2	8.5	26	59
C_A mol/l	0.366	1.46	2.78	6.22	10.9

a, $N_A = k_G (P_{AG} - P_{Ai}) = k_L (C_{Ai} - C_{AL}) \Rightarrow$

$$\frac{k_G}{k_L} = \frac{C_{Ai} - C_{AL}}{P_{AG} - P_{Ai}} \Rightarrow C_{Ai} = 0.0359(4001 - P_{Ai}) + 0.55 E^{-3}$$

Vid fasgränsytan står de två faserna i jämvikt

interpolation : $C_{Ai} = 0.687 \Rightarrow P_{Ai} = 1.38$ mm Hg

plot : $C_{Ai} = 0.685 \Rightarrow P_{Ai} = 1.6$ mm Hg

b, P_A^* jämviktstryck av SO_2 då $C_A = C_{AL} = 0.55$

C_A^* jämvik~~ts~~skoncentration av SO_2 då $P_A = P_{AG} = 30$ mmHg

jämviktskurva $\Rightarrow P_A^* = 120 \text{ Pa} \quad C_A^* = 7$

$$K_G = \frac{N_A}{P_{AG} - P_A^*} = \frac{k_G (P_{AG} - P_{Ai})}{P_{AG} - P_A^*} = 3.88 E^{-1}$$

$$K_L = \frac{k_L (C_{Ai} - C_{AL})}{C_A^* - C_{AL}} = 2,33 E^{-6}$$

c,

$$\frac{\text{motstånd i gasfas}}{\text{totalt motstånd}} = \frac{1/k_G}{1/K_G} = 0,977 \approx 98\%$$

21.4}

$K_L = 4.2\ E^{-6}\ m/s$

$C_{NH_3L} = 4\ E^3\ mol/m^3$

$P_{NH_3G} = 0.03\ atm$

$P_{Ai} = 0.0134 \cdot C_{Ai} \Rightarrow m = 0.0134\ atm/_{kmol\ m^3}$

$k_G^{-1}/K_G^{-1} = 0.75$

a,

$$\left.\begin{array}{l} K_G^{-1} = k_G^{-1} + \dfrac{m}{k_L} \\ K_L^{-1} = k_L^{-1} + (mk_G)^{-1} \end{array}\right\} K_G^{-1} = k_G^{-1} + m\left(K_L^{-1} - (mk_G)^{-1}\right)$$

$$\Rightarrow K_G = \frac{K_L}{m} = \underline{\underline{3.13\ E^{-4}}}\ kmol/m^2\,s\,atm$$

b,

$$k_G = K_G/_{0.75} = \underline{\underline{4.18\ E^{-4}}}\ kmol/m^2\,s\,atm$$

c,

$$k_G(P_{NH_3G} - P_{NH_3i}) = k_L(C_{NH_3i} - C_{NH_3L})$$

$$k_L^{-1} = K_L^{-1} - (mk_G)^{-1} \Rightarrow k_L = 1.68\ E^{-5}$$

$$P_{NH_3i} = m \cdot C_{NH_3i}$$

$$\Rightarrow (k_L + k_G m)\,C_{NH_3i} = k_G P_{NH_3G} + k_L C_{NH_3L}$$

$$\Rightarrow C_{NH_3i} = \underline{\underline{3.56}}\ kmol/_{m^3} \Rightarrow P_{NH_3i} = \underline{\underline{0.048\ atm}}$$

21.5 }

$K_L = 2{,}5\,E^{-3}$ m/s

$k_L = 8{,}5\,E^{-4}$ m/s

$$\frac{k_L^{-1}}{K_L^{-1}} = 3$$

$$K_L^{-1} = k_L^{-1} + (mk_G)^{-1}$$

$$\frac{k_L^{-1}}{(mk_G)^{-1} + k_L^{-1}} = 3$$

⇒ Större motstånd i vätskefasen än totalt

⇒ Orealistiskt !

21.6}

$m = 3.87 \ \mathrm{bar/mol\ H_2S/m^3\,vätska}$

$P_{AL} = 195 \ \mathrm{mmHg} = 0.259935 \ \mathrm{bar}$

$C_{Ai} = 0.0525 \ \mathrm{mol/m^3}$

$k_G = 3 \ \mathrm{mol/m^2 s\,bar}$

$K_G = 2.33 \ \mathrm{mol/m^2 s\,bar}$

a, $N_A = k_G(P_{AG} - P_{Ai})$

$P_{Ai} = m \cdot C_{Ai} = 0.203175 \ \mathrm{bar}$

$\Rightarrow N_A = 0.17028 \ \mathrm{mol/m^2 s}$

b, $C_{AL} = \dfrac{P_A^*}{m}$

$N_A = K_G(P_{AG} - P_A^*) \Rightarrow P_A^* = P_{AG} - \dfrac{N_A}{K_G} = 0.186853454 \ \mathrm{bar}$

$\Rightarrow C_{AL} = 0.048282546 \ \mathrm{mol/m^3}$

c, Gasfilm : $\dfrac{k_G^{-1}}{K_G^{-1}} = 0.7766$

vätskefilm : $K_L = mK_G = 9.0171 \ \mathrm{m/s}$

$k_L^{-1} = K_L^{-1} - (mk_G)^{-1} \Rightarrow k_L = 40.37507463 \ \mathrm{m/s}$

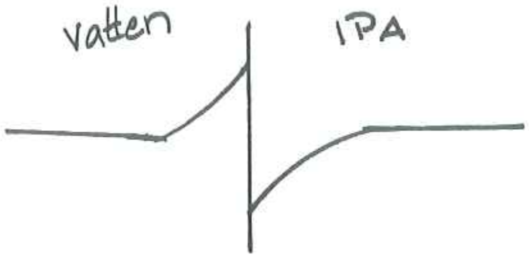
$\dfrac{k_L^{-1}}{K_L^{-1}} = 0.2233$

Motståndet i gasen är störst

d, P_{AL} = konstant

$$C_A = \frac{P_{AL}}{m} = 0.0671667 \text{ mol/m}^3$$

22.1



A = IPA fasen

B = vattenfasen

C = propionsyra

$C_{GA} = 0.05 \ kmol/m^3$

$C_{GB} = 0.05 \ kmol/m^3$

$k_{cB} = 5E^{-5} \ m/s$; vattenfasen

$k_{cA} = 1E^{-4} \ m/s$; IPAfasen

$M_{IPA} = 60 \ g/mol$

$$\frac{X_{iH_2O}}{X_{iIPA}} = 1.5$$

$$\rho_{IPA} = 790 \ kg/m^3$$

$$C_i = \frac{\rho_i}{M_i}$$

$$\left.\begin{aligned} N_c &= k_{cB}(C_{ciB} - C_{cB}) \\ N_c &= k_{cA}(C_{cA} - C_{ciA}) \end{aligned}\right\} \quad k_{cB}(C_{ciB} - C_{cB}) = k_{cA}(C_{cA} - C_{ciA}) \quad (*)$$

$$X_{iIPA} = \frac{X_{iH_2O}}{1.5} \qquad X_i = \frac{C_i}{C_T} \qquad C_i = \frac{\rho_i}{M_i}$$

$$\frac{C_{iIPA}}{C_{TIPA}} = \frac{C_{iH_2O}}{C_{TH_2O} \cdot 1.5} \qquad C_{iA} = \frac{C_{iB} \cdot \frac{790}{6E^{-3}}}{1.5 \cdot \frac{1000}{18E^{-3}}}$$

$$\Rightarrow C_{iA} = 0.158 \, C_{iB}$$

$$\Rightarrow (*) \ k_{cB}(C_{ciB} - C_{LB}) = k_{cA}(C_A - 0.158 C_{iB})$$

$$\Rightarrow C_{iB} = 0.114 \ kmol/m^3$$

$$N_A = k_{cB}(C_{iB} - C_{cB}) = \underline{\underline{3.2E^{-3}}} \ mol/m^2 s$$

22.2}

$Q = 0.5\ l/min$

$\phi = 4mm$

$H = 40cm$

$h = 5000\ W/m^2K$

$C_l = 240\ molO_2/m^3H_2O$

$C_\infty = 230\ molO_2/m^3H_2O$

$T = 298K$

$\rho_{H_2O} = 997.1\ kg/m^3$

$C_{P_{H_2O}} = 4.178\ kJ/kgK$

$\nu_{H_2O} = 0.896\ E^{-6}\ m^2/s$

$P_r = 6.18$

a, allt motstånd i vätskefasen & konstant syrehalt i bubblorna:

$$N_A = k_c(c_l - C_\infty)$$

$$\frac{h}{\rho C_p} P_r^{2/3} = k_c S_c^{2/3} \Rightarrow k_c = 6.9E^{-5}\ m/s$$

$$\Rightarrow N_A = \underline{\underline{6.9E^{-4}}}\ mol/m^2s$$

b, 1, Ta reda på hur lång tid bubblorna är i vattnet

2, Ta reda på antal bubblor

$$V_{max} = \left(\frac{4\Delta\rho g D_p}{3\rho_l C_D}\right)^{1/2}$$

$$Re = \frac{V_{max} D}{\nu}$$

$$C_D = C_D(Re)$$

Gissa $C_D \Rightarrow V_{max} \Rightarrow Re \Rightarrow C_D$

$\Rightarrow V_{max} = 0.323$ m/s

stig tid $= \frac{H}{V_{max}} = 1.23667$ s

$Q_l = 0.5$ l/min $= 8.333 E^{-6}$ m³/s

$\frac{4\pi r^3}{3Q} = 248.679$ bubblor/s

$\Rightarrow 1.236 \cdot 248.679 = 307.5$ bubblor samtidigt i akvariet

Bubblornas totala area $= \overset{\frac{4\pi D^2}{4}}{\pi D^2} \cdot 307.5 = 0.01546$ m²

Totalt flöde: $N_A \cdot A = 1.0666 E^{-5}$ mol/s

Antal fiskar: $\frac{\text{Tillgängligt}}{\text{Förbrukning}} = \frac{1.1 E^{-5}}{5 E^{-6}} \approx \underline{\underline{2 \text{ fiskar}}}$

22.3}

$D_{AB} = E^{-8}\ m^2/s$

$t = 1800\ s$

$\omega_A = 0{,}0001$

$\omega_{AS} = 0{,}05$

$\omega_{A0} = 0$

låga koncentrationer ⇒

$$\frac{\omega_{AS} - \omega_A}{\omega_{AS} - \omega_{A0}} = \frac{C_{AS} - C_A}{C_{AS} - C_{A0}} = 0{,}998$$

Semiinfinit medium

$$\frac{C_{AS} - C_A}{C_{AS} - C_{A0}} = \operatorname{erf}\left(\frac{z}{2\sqrt{D_{AB} \cdot t}}\right)$$

$$\Rightarrow \frac{z}{2\sqrt{D_{AB} \cdot t}} = 2{,}2$$

$$\Rightarrow \quad z = \underline{\underline{0{,}0187\ m}}$$

22.4)

$V = 3 m^3$

$D = 4 cm$

$C_{Akr} = 0{,}05 \ mol/m^3$

$P = 1 atm$

$T = 298 K$

$P_s^o = 5 mm Hg = 666.5 Pa$

$h = 34 \ W/m^2K$

$$C_s = \frac{P_s^o}{RT} = 0.269$$

$N_A = k_c (C_{As} - C_{A\infty})$ men $C_{A\infty}$ varierar med tiden

$$\frac{dC_{A\infty}}{dt} = \frac{N_A \cdot A}{V}$$

Chilton - Colburn: $j_H = j_D \Rightarrow \frac{h}{\rho C_p} Pr^{2/3} = k_c Sc^{2/3}$

$\Rightarrow k_c = 0.01211689$

glöm ej att det är 10st "bollar"

$$\Rightarrow \frac{dC_{A\infty}}{dt} = k_c \cdot A (C_s - C_{A\infty})$$ integrering $\Rightarrow$

$$\ln \left(\frac{0{,}269 - 0}{0{,}269 - C_{Akr}} \right) = 2{,}03020457 E^{-4} \cdot t$$

$\Rightarrow t = 1012{,}845 = 16.88 min$

Svar: Malen dör efter 16,9 min

22.5 } $P_{AL} = 0.15\,atm$ $N_{AL} = 4\,E^{-5}\,mol/m^2s$

$C_{AL} = 1\,mol/m^3$ $k_{GL} = 4E^{-4}\,mol/m^2s\,atm$

Henry's lag : $P_i = x_i \cdot H = C_i \tilde{H}$ $\tilde{H} = \frac{H}{C_T}$

$$\left.\begin{array}{l} C_i = 1\,mol/m^3 \\ P_i = 0{,}03\,atm \end{array}\right\} \Rightarrow \tilde{H} = \frac{P_i}{C_i} = 0{,}03$$

a, $N_A = k_G(P_{AG} - P_{Ai}) \Rightarrow P_{AG} - P_{Ai} = \underline{\underline{0.1\ atm}}$

$C_{Ai} = \frac{P_{Ai}}{\tilde{H}} = \frac{P_{AG} - 0.1}{\tilde{H}} = 1{,}6667 \Rightarrow C_{Ai} - C_{AL} = \underline{\underline{0.6667}}\,mol/m^3$

$N_A = k_L(C_{Ai} - C_{AL}) \Rightarrow k_L = \underline{\underline{6\,E^{-5}}}$

$P_A^* = \tilde{H} C_{AL} = 0.03\ atm \Rightarrow P_{AG} - P_A^* = \underline{\underline{0.12\,atm}}$

$N_A = K_G(P_{AG} - P_A^*) \Rightarrow \underline{\underline{3.33\,E^{-4}}} = K_G$

$C_A^* = P_{AG}\tilde{H}^{-1} = 5\,mol/m^3 \Rightarrow C_A^* - C_{AL} = \underline{\underline{4\,mol}}/m^3$

$N_A = K_L(C_A^* - C_{AL}) \Rightarrow K_L = \underline{\underline{1\,E^{-5}}}\ m/s$

b, $\frac{\text{motstånd i gasfilm}}{\text{totalt motstånd}} = \frac{k_G^{-1}}{K_G^{-1}} = 0{,}8325 \approx 83{,}3\%$